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Comparative Analysis of the MCDM Methods with Multiple Normalization Techniques: Three Hybrid Models Combine MPSI with DNMARCOS, AROMAN, and MACONT Methods

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Abstract: *Several multi-criteria decision-making methods have been developed to solve complex decision problems encountered in business and daily life. These methods offer a systematic approach to evaluating multiple decision alternatives and conflicting criteria. The normalization stage in the multi-criteria decision-making process is important in evaluating the contribution of criteria or alternatives to the process in a fair, consistent, comparable, and objective way. Various methods employ one or more normalization techniques, and the combined use of multiple normalization techniques allows for a comprehensive analysis. In this study, Double Normalized Measurement of Alternatives and Ranking According to COmpromise Solution (DNMARCOS), Alternative Ranking Order Method Accounting for Two Step Normalization (AROMAN), and Mixed Aggregation by Comprehensive Normalization Technique (MACONT) methods used multiple normalization techniques are compared and evaluated for a robot vacuum cleaner selection problem. The relations of the ranking results were evaluated by correlation analysis. The performance comparisons of the methods were made in terms of the final scores' standard deviations and the methods' computational complexity. The findings indicate that DNMARCOS has the best performance among the three methods and MACONT has the lowest performance.*

Keywords: Multi-criteria Decision Making, Normalization Techniques, MPSI, DNMARCOS, AROMAN, MACONT

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1. Introduction

Many simple and complex decision problems are encountered in our daily life and professional business life. While most decision problems are solved instantly by the brain's own mechanisms, some decisions take longer. Many factors, such as the number of criteria to be considered in the decision problem, the relative or absolute importance levels and priorities of the criteria, the contradictory aspects of the criteria, the number of alternatives to be chosen from, the degree of impact of the decision, the element of uncertainty, the attitude of the decision maker make it difficult to make some decisions. In this way, many qualified decision-making methods have been developed to help solve complex decision problems. The literature, which has been enriched with diversified methods in recent years, has divided multi-criteria decision problems into two categories: multi-objective and multi-attribute (Tzeng & Huang, 2011: 1). Multi-objective decision problems can be defined as decision problems that involve more than one goal (objective) that needs to be optimized. On the other hand, multi-attribute decision-making techniques are used for

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prioritizing and weighting the criteria considered in decision problems and/or evaluating and ranking multiple alternatives with multiple criteria in the same direction or conflicting with each other. In this study, the term multi-criteria decision-making (MCDM) is used to mean multi-attribute decision-making.

In many MCDM problems, the criteria have different scales. Therefore, preprocessing is necessary to achieve a consistent scale, allowing the aggregation of numerical and comparable criteria to obtain a final score for each alternative (Vafaei et al., 2018). Normalization is an important step in the multi-criteria decision-making process that makes the evaluation criteria considered in the decision problem dimensionless and unitless (Jahan & Edwards, 2014). An increasing number of methods in the literature differ regarding their normalization and criteria aggregation methods. Due to these differences, the final rankings obtained from different methods may vary and be similar. The normalization process is vital and performed to standardize criteria with varying units of measurement and make them comparable and evaluable together. There are different functions proposed in the literature for this process. The functions vary according to whether the evaluated criterion is benefit or cost oriented. The most used normalization methods are given in the following equations (Aytekin, 2021: 4-5; Kosareva et al., 2018; Vafaei et al., 2018).

Linear Sum Based Normalization :

Benefit Criteria $$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad (1)$$

Cost Criteria $$n_{ij} = \frac{1/x_{ij}}{\sum_{i=1}^{m} 1/x_{ij}} \quad (2)$$

Linear Ratio Based Normalization :

Benefit Criteria $$n_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \quad (3)$$

Cost Criteria $$n_{ij} = \frac{\min_i x_{ij}}{x_{ij}} \quad (4)$$

Linear Max-Min Normalization :

Benefit Criteria $$n_{ij} = \frac{x_{ij} - min\, x_{ij}}{\max x_{ij} - \min x_{ij}} \quad (5)$$

Cost Criteria $$n_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}} \quad (6)$$

Vector Normalization :

Benefit Criteria $$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad (7)$$

Cost Criteria $$n_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad (8)$$

As seen in the equations, different normalization techniques have different calculation procedures. The normalization technique chosen in the decision-making process significantly impacts the final decision (Baydaş et al., 2023; Jafaryeganeh et al., 2020; Polska et al., 2021). For this reason, the normalization stage is an important stage that diversifies and differentiates multi-criteria decision-making methods. A review of multi-criteria decision-making techniques using these methods is presented in Table 1. The table shows that the most used normalization functions among multi-criteria decision-making methods are linear ratio-based functions and linear max-min normalization functions.

The ease of understanding and application of linear techniques can be shown as two of the reasons why they are highly preferred in multi-criteria decision-making methods. On the other hand, the fact that the number of decision units evaluated in decision problems is generally limited, the data set is not required to be normally distributed, and the main purpose is to facilitate the comparability of variables by removing measurement units and extreme values can be listed among the reasons why linear normalization methods are used more frequently.

Table 1. Normalization Techniques Used by Different MCDM Methods

	Linear Sum Based Norm.	Linear Ratio Based Norm.	Linear Max-Min Norm.	Vector Norm.
TOPSIS				✓
VIKOR			✓	
ELECTRE		✓		✓
ARAS	✓			
COPRAS	✓			
MAUT			✓	
MAIRCA			✓	
CoCoSo			✓	
CILOS	✓			
SECA		✓		
MEREC		✓		
LOPCOW			✓	
CRADIS		✓		
CODAS		✓		
WASPAS		✓		
MOORA				✓
MOOSRA				✓
TODIM			✓	
MABAC			✓	
EDAS		✓		
MARA		✓		
TARO		✓		
MARCOS		✓		
WEDBA		✓		
KEMIRA-M			✓	

Note: TOPSIS (Technique for Order Preference by Similarity to Ideal Solution); VIKOR (Vlse Kriterijumska Optimizacija I Kompromisno Resenje-), ELECTRE (ÉLimination Et Choix Traduisant la REalité), ARAS (Additive Ratio Assessment), COPRAS (Complex. Proportional Assessment), MAUT (Multiple Attribute Utility Theory), MAIRCA (Multi Atributive Ideal-Real Comparative Analysis), CoCoSo (Combined Compromise Solution), CILOS (Criterion Impact Loss), SECA (Simultaneous Evaluation of Criteria and Alternatives), MEREC (Method based on the Removal Effects of Criteria), LOPCOV (Logarithmic Percentage Change-driven Objective Weighting), CRADIS (Compromise Ranking of Alternatives from Distance to Ideal Solution), CODAS (Combinative Distance-Based Assessment), WASPAS (Weighted Aggregated Sum Product Assessment), MOORA (Multi-Objective Optimization method on the basis of Ratio Analysis), MOOSRA (Multi-Objective Optimization on the Basis of Simple Ratio Analysis), TODIM (The Interactive Multi-Criteria Decision Making), MABAC (Multi-Attributive Border Approximation Area Comparison), EDAS (Evaluation based on Distance from Average Solution), MARA (Magnitude of the Area for the Ranking of Alternatives), TARO (Technique of Accurate Ranking Order), MARCOS (Measurement of Alternatives and Ranking according to Compromise Solution), WEBDA (Weighted Euclidean Distance Based Approximation), and KEMIRA-M (Modified Kemeny Median Indicator Ranks Accordance).

Since the data normalization stage is one of the most important stages that differentiates the methods from each other, studies on the comparison of methods using different normalization functions and the fact that different final weights and rankings can be obtained because of applying the same method using different normalization functions have been exemplified by various authors recently. Lakshm and Venkatesan (2014) applied linear max-min, linear sum-based, linear max, and Gaussian normalization techniques separately to the TOPSIS method using vector normalization in its original form. Then, the evaluation comparisons were made regarding the algorithm's time and space complexity. According to the results obtained, the linear sum-based normalization technique was found to be the most advantageous method in terms of time and space complexity, while the linear max-min normalization technique was ranked last. Özdağoğlu (2014) tested the effects of linear, vector, and non-monotone normalization techniques on MOORA method results with synthetic data sets produced under different scenarios. According to the results, it is seen that the differences in the results obtained, especially in non-monotone normalization, are more significant, while the results of other techniques are very similar. Vafei et al. (2016) replicated the AHP method using five different normalization techniques. They found that the logarithmic normalization technique gave the most different results from the others (linear sum, linear max-min, linear interval, and vector normalization). Vafei et al. (2016) repeated the Analytic Hierarchy Process (AHP) method using five different normalization techniques and found that the logarithmic normalization technique gave the most different result from the others (linear sum, linear max-min, linear interval, and vector normalization). Jafaryeganeh et al. (2020) examined the effect of maximum linear, linear max-min, and vector normalization techniques on the Weighted Sum Model (WSM), TOPSIS, and ELECTRE rankings. They concluded that the rankings for different normalization techniques are highly correlated in all three multi-criteria decision-making methods. Satıcı (2021) presented a comparative analysis of the WASPAS method using a linear ratio-based 0-1 Interval Normalization technique with other linear normalization techniques (max, max-min, sum-based), vector normalization, logarithmic normalization, and accuracy-enhanced normalization techniques. When the results obtained are examined, it is found that the results of the normalization technique initially used for the WASPAS method and the linear sum-based normalization and vector normalization techniques are very close to each other so that these techniques can be used interchangeably in the WASPAS method. The Linear Normalization (Max-Min) technique gives less similar results compared to other normalization techniques. Vafaei et al. (2022) perform a similar study with Simple Additive Weighting (SAW) method. They used linear ratio, linear sum, linear max-min, and vector normalization techniques separately in their study, investigating how SAW ranking results are affected by the normalization technique used. It was seen that the most different ranking result from the other three was obtained with the linear max-min normalization technique, while the results of linear summation and vector normalization were closer to each other. Another study using multiple normalization was recently conducted by Puška et al. (2023). They used two different linear normalization techniques together in both the MEREC method for criteria weighting and the CRADIS method for ranking in the electric vehicle selection problem and performed the integration with the arithmetic mean method.

In addition to studies aiming to show how the results of methods known to use a single normalization technique are affected by different normalization techniques, specific multi-criteria decision-making methods that use more than one normalization technique together and offer integrated single-ranking forms have started to be developed to obtain more reliable and robust results. The MACONT method, one of the methods with the mentioned features, was introduced by its developers, Wen et al., in 2020 as an algorithm that simultaneously uses three different linear normalization techniques (sum-based, ratio-based, and max-min). In his study in 2023, Nguyen introduced new variations to the method with different combinations of normalization techniques. In the literature, it is seen that studies on the MACONT method have been carried out in areas such as insulation material evaluation (Aksakal et al., 2022), tradeoff analysis in transportation budget allocation (Truong & Li, 2023), waste management (Simic et al., 2023), vocational education quality assessment (Huang & Chen, 2023).

The Double Normalized MARCOS (DNMARCOS) method was developed by Ivanović et al. in 2022. In this method, linear and vector normalization techniques are applied together. Three different methods (Complete compensatory model (CCM), Un-compensatory model (UCM), and Incomplete compensatory model (ICM)) are used to obtain integrated utility values from normalized values. A single final ranking score is obtained with a weighted average function that integrates these utility values (Ivanović et al., 2022). DNMARCOS, which is still a new method, was used by its developers to select a truck mixer concrete pump, and then Saha et al. (2023) used it to solve the warehouse site selection problem.

Alternative Ranking Order Method Accounting for Two-Step Normalization (AROMAN), a newer method than DNMARCOS, was developed in Bošković et al (2023a). The method, which uses linear max-min normalization function and vector normalization together, achieves the result with fewer computational steps and calculations than the DNMARCOS method. Immediately after the method was developed in 2023, it started to deepen its place in the literature by being used for different purposes such as improving the sustainability of different postal networks (Nikolić et al., 2023), selecting the type of cargo bicycle delivery (Bošković et al., 2023b), selecting professional drivers (Čubranić-Dobrodolac et al., 2023).

In this study, DNMARCOS, AROMAN, and MACONT methods, which use different multiple normalization techniques together, are discussed comparatively based on the Modified Preference Selection Index (MPSI), and the method applications are carried out on the robot vacuum cleaner selection problem. In addition to these methods, CRADIS, MAUT, and MOOSRA methods, which use only one of the normalization techniques, were also analyzed, and the results were evaluated comparatively. In this context, the data set used is introduced in the second part of the study, and detailed information about the applied methods is given. The findings and results of the comparative analyses are presented in the third section, and the conclusions are evaluated in the last section.

2. Methodology

2.1. Data Set

This research used a dataset consisting of 41 alternatives and 11 criteria. To determine the list of alternatives, the names of all the brands of robot vacuum cleaners available for purchase in Turkey (60 brands) were first listed. After that, the most well-known brands in the market were determined with the help of 100 participants. The models of the most recognized 10 brands in sales were included in the alternative list. Evaluation criteria were decided based on product features and factors that are commonly considered while making purchases. These criteria are shown in Table 2.

Table 2. Criteria of the Research

Code	Criterion	Unit	Optimization Direction
C1	Price	TL	Min
C2	Suction	Pa	Max
C3	Passable Obstacle Level	mm	Max
C4	Hopper Capacity	ml	Max
C5	Charging Time	hour	Min
C6	Noise Level	Dba	Min
C7	Number of Cleaning Modes	number	Max
C8	Weight	kg	Min
C9	Height	Mm	Min
C10	Product Review	point	Max
C11	Wireless Working Time	second	Max

The dataset was created using information obtained from an online shopping platform and brand websites between 04.08.2023 and 09.08.2023. The dataset used in the analysis is presented in Appendix 1.

2.2. Modified Preference Selection Index (MPSI) Method

The Preference Selection Index (PSI) Method is one of the objective weighting methods developed by Maniya and Bhatt in 2010 to solve multicriteria decision-making problems. Then Gligori´c et al. (2022) modified the method. The method is based on calculating the degree of oscillation using the Euclidean distance between the normalized values of the criteria and their mean values. The method's strengths are that it provides an objective calculation of criterion weights, is easy to understand, and is not time-consuming. The MPSI method has five steps for calculating the criteria weights (Gligori´c et al., 2022).

Step 1: Construct the initial decision-making matrix (A/C)

The initial decision-making matrix is constructed where m denotes alternatives and n denotes criteria as follows.

$$(A/C) = [x_{ij}]_{m\times n} = \begin{vmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{vmatrix} \quad (9)$$

Step 2: Obtain the normalized decision matrix R.

Linear normalization technique normalizes the initial decision matrix elements, and normalized matrix R is formed as follows. Equations 3 and 4 are used to calculate the normalized values (r_{ij}) in the matrix R.

$$R = [r_{ij}]_{m\times n} = \begin{vmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{vmatrix} \quad (10)$$

Step 3: Calculate each criterion's mean normalized value v alt j.

$$v_j = \frac{\sum_{i=1}^{m} r_{ij}}{m}, \quad j = 1, 2, \dots, n \quad (11)$$

Step 4: Calculate the preference variation value (p_j) of criteria.

$$p_j = \sum_{i=1}^{m} (r_{ij} - v_j)^2, \quad j = 1, 2, \dots, n \quad (12)$$

Step 5: Calculate the criteria weights (w_j).

$$w_j = \frac{p_j}{\sum_{j=1}^{n} p_j} \quad (13)$$

2.3. Double Normalized MARCOS (DNMARCOS) Method

MARCOS method presented by Stević et al. (2020) is based on defining the relationship between alternatives and ideal- anti-ideal values of the alternatives. The original MARCOS method used ratio-based linear normalization functions to make the criteria unidimensional. Ivanovic et al. suggested the double normalized MARCOS (DNMARCOS) method in 2022. Linear and vector normalization are utilized in the DNMARCOS method. The method consists of three types of aggregation models, specifically the complete compensatory model (CCM), the Un-compensatory model (UCM), and the Incomplete compensatory model (ICM), to aggregate the different normalization results. The steps for the DNMARCOS algorithm are as follows (Ivanovic et al, 2022):

Step 1: Build the initial decision matrix M

$$M = [x_{ij}]_{m\times n} \quad (14)$$

Step 2: Normalize the matrix M by using the simple linear normalization method given in Equations 3 – 4 and obtain the M' matrix.

$$M' = [x'_{ij}]_{m\times n} = \begin{vmatrix} x'_{11} & \cdots & x'_{1n} \\ \vdots & \ddots & \vdots \\ x'_{m1} & \cdots & x'_{mn} \end{vmatrix} \quad (15)$$

Step 3: Obtain the vector normalized matrix (VNM) M'' by using Equations 7-8.

$$M'' = \left[x''_{ij}\right]_{m\times n} \tag{16}$$

Step 4: Built the extended linear normalized matrix (ELNM) Θ'

Add the ideal (ID) and anti-ideal (AID) solutions to the linear normalized matrix M' as rows.

$$x_j'^{(+)} = \begin{cases} \max\limits_i x'_{ij}, & for\ maximization\ criteria \\ \min\limits_i x'_{ij}, & for\ minimization\ criteria \end{cases} \tag{17}$$

$$x_j'^{(-)} = \begin{cases} \min\limits_i x'_{ij}, & for\ maximization\ criteria \\ \max\limits_i x'_{ij}, & for\ minimization\ criteria \end{cases} \tag{18}$$

Step 5: Derive the extended vector normalized matrix Θ''

Add the ideal (ID) and anti-ideal (AID) solutions to the vector normalized matrix M'' as rows.

$$x_j''^{(+)} = \begin{cases} \max\limits_i x''_{ij}, & for\ maximization\ criteria \\ \min\limits_i x''_{ij}, & for\ minimization\ criteria \end{cases} \tag{19}$$

$$x_j''^{(-)} = \begin{cases} \min\limits_i x''_{ij}, & for\ maximization\ criteria \\ \max\limits_i x''_{ij}, & for\ minimization\ criteria \end{cases} \tag{20}$$

Step 6: Multiply the ELNM with criteria weights and obtain Γ' matrix.

$$\tilde{a}_{ij} = w_j x'_{ij} \tag{21}$$

$$\tilde{a}_j^{(-)} = w_j x_j'^{(-)} \tag{22}$$

$$\tilde{a}_j^{(+)} = w_j x_j'^{(+)} \tag{23}$$

Step 7: Calculate the weighted EVNM matrix Γ'' by using following equations.

$$\tilde{\beta}_{ij} = \left(a''_{ij}\right)^{w_j} \tag{24}$$

$$\beta_{ij}^{(-)} = \left(a_j''^{(-)}\right)^{w_j} \tag{25}$$

$$\beta_{ij}^{(+)} = \left(a_j''^{(+)}\right)^{w_j} \tag{26}$$

Step 8: Compute the sub-ordinate values of the alternatives.

Sub-ordinate values are determined based on the CCM, UCM and ICM models.

8.1. Sub-ordinate values of alternatives by CCM model

$$S_i^{(1)} = \sum_{j=1}^{n} \tilde{a}_{ij}, \quad (i = 1,2,\dots,m) \tag{27}$$

$$AID_i^{(1)} = \sum_{j=1}^{n} x_{ij}^{(-)}, \quad (i = 1, 2, \dots, m) \tag{28}$$

$$ID_i^{(1)} = \sum_{j=1}^{n} x_{ij}^{(+)}, \quad (i = 1,2, \dots, m) \tag{29}$$

8.2. Sub-ordinate values of alternatives by UCM model

$$S_i^{(2)} = \max_j \ (w_j \times (1 - x'_{ij}), \quad (i = 1, 2, \dots, m) \tag{30}$$

$$AID_i^{(1)} = \max_j \ (w_j \times (1 - x_{ij}^{(-)}), \quad (i = 1, 2, \dots, m) \tag{31}$$

$$ID_i^{(2)} = \max_j \ (w_j \times (1 - x_{ij}^{(+)}), \quad (i = 1, 2, \dots, m) \tag{32}$$

8.2. Sub-ordinate values of alternatives by ICM model

$$S_i^{(3)} = \prod_{j=1}^{n} \tilde{\beta}_{ij}, \quad (i = 1, 2, \dots, m) \tag{33}$$

$$AID_i^{(3)} = \prod_{j=1}^{n} \beta_{ij}^{(-)}, \quad (i = 1, 2, \dots, m) \tag{34}$$

$$ID_i^{(3)} = \prod_{j=1}^{n} \beta_{ij}^{(+)}, \quad (i = 1, 2, \dots, m) \tag{35}$$

9. Calculate the utility degree of alternatives based on CCM, UCM, and ICM models.

9.1. Utility degree of alternatives based on CCM

$$h_i^{1(-)} = \frac{S_i^{(1)}}{AID_i^{(1)}} \tag{36}$$

$$h_i^{1(+)} = \frac{S_i^{(1)}}{ID_i^{(1)}} \tag{37}$$

9.2. Utility degree of alternatives based on UCM

$$h_i^{2(-)} = \frac{S_i^{(2)}}{AID_i^{(2)}} \tag{38}$$

$$h_i^{2(+)} = \frac{S_i^{(2)}}{ID_i^{(2)}} \tag{39}$$

9.3. Utility degree of alternatives based on ICM

$$h_i^{3(-)} = \frac{S_i^{(3)}}{AID_i^{(3)}} \tag{40}$$

$$h_i^{3(+)} = \frac{S_i^{(3)}}{ID_i^{(3)}} \tag{41}$$

10. Compute the utility values of alternatives based on CCM, UCM, and ICM models.

10.1. Utility values of alternatives based on CCM

$$\lambda_i^{1(-)} = \frac{h_i^{1(+)}}{h_i^{1(+)} + h_i^{1(-)}} \tag{42}$$

$$\lambda_i^{1(+)} = \frac{h_i^{1(-)}}{h_i^{1(+)} + h_i^{1(-)}} \tag{43}$$

10.2. Utility values of alternatives based on UCM

$$\lambda_i^{2(-)} = \frac{h_i^{2(+)}}{h_i^{2(+)} + h_i^{2(-)}} \tag{44}$$

$$\lambda_i^{2(+)} = \frac{h_i^{2(-)}}{h_i^{2(+)} + h_i^{2(-)}} \tag{45}$$

10.3. Utility values of alternatives based on ICM

$$\lambda_i^{3(-)} = \frac{h_i^{3(+)}}{h_i^{3(+)} + h_i^{3(-)}} \tag{46}$$

$$\lambda_i^{3(+)} = \frac{h_i^{3(-)}}{h_i^{3(+)} + h_i^{3(-)}} \tag{47}$$

Step 11: Obtain the general utility value for each alternative.

$$\lambda_i = p \times \left(\frac{h_i^{1(+)} + h_i^{1(-)}}{1 + \frac{1-\lambda_i^{1(-)}}{\lambda_i^{1(-)}} + \frac{1-\lambda_i^{1(+)}}{\lambda_i^{1(+)}}} \right) + q \times \left(\frac{h_i^{2(+)} + h_i^{2(-)}}{1 + \frac{1-\lambda_i^{2(-)}}{\lambda_i^{2(-)}} + \frac{1-\lambda_i^{2(+)}}{\lambda_i^{2(+)}}} \right) + (1-p-q) \times \left(\frac{h_i^{3(+)} + h_i^{3(-)}}{1 + \frac{1-\lambda_i^{3(-)}}{\lambda_i^{3(-)}} + \frac{1-\lambda_i^{3(+)}}{\lambda_i^{3(+)}}} \right) \tag{48}$$

Step 12: Rank the alternatives and determine the best one. Maximum general utility value shows the best alternative.

2.4. Alternative Ranking Order Method Accounting for Two Step Normalization (AROMAN)

The AROMAN method, developed by Bošković et al. (2023a), is a MCDM method in which the matrices obtained from the two-stage normalization process are integrated by calculating the average. The method, which aims to obtain more robust ranking results by applying linear and vector normalization techniques together, was first applied for the selection of electric vehicles. The method, which is still very new in the literature, was later applied to cargo bicycle delivery mode selection (Boskovic et al., 2023b), sustainable human resource management assessment (Rani et al., 2023), driver selection (Čubranić-Dobrodolac et al., 2023), assessment of sustainability of rural postal network (Nikolić et al., 2023), wastewater treatment technology selection (Alrasheedi et al., 2024), sustainable delivery model selection (Dobrodolac et al., 2024), sustainable competitiveness assessment (Kara et al., 2024).

AROMAN method has six computational steps as following (Boskovic et al., 2023a);

Step 1: Initial decision matrix $X = \left[x_{ij}\right]_{m\times n}$ is determined with m alternative n criteria.

Step 2: In the second step, the initial decision matrix X is normalized by linear max-min (Equation 5 and 6) and vector normalization (Equation 7 and 8) methods. Same normalization equations are used both benefit and cost criterion types. Then aggregated average normalization values (t_{ij}^{norm}) are calculated by Equation 49.

$$t_{ij}^{norm} = \frac{\beta t_{ij} + (1-\beta)t_{ij}^{*}}{2} \qquad i = 1, 2, \dots, m;\ j = 1, 2, \dots, n \tag{49}$$

β denotes the weight of the linear normalization method, varies between 0 and 1.

Step 3: Weighted normalized matrix is obtained by the help of Equation 50.

$$\hat{t}_{ij} = W_{ij}\ .t_{ij}^{norm} \qquad i = 1, 2, \dots, m;\ j = 1, 2, \dots, n \tag{50}$$

Step 4: The sum weighted normalized values of criteria are calculated by separating their types (max and min).

$$L_i = \sum_j \hat{t}_{ij}^{(\min)} \qquad i = 1, 2, \dots, m;\ j \in cost\ criteria \tag{51}$$

$$A_i = \sum_j \hat{t}_{ij}^{(\max)} \qquad i = 1, 2, \dots, m;\ j \in benefit\ criteria \tag{52}$$

Step 5. The final values of criteria function (R_i) are computed.

$$R_i = L_i^{\lambda} + A_i^{(1-\lambda)} \qquad i = 1, 2, \dots, m \tag{53}$$

Step 6. Rank the R_i values by decreasing order. The highest value of R_i denotes the best alternative.

2.5. MACONT-T6 Method

MACONT is a MCDM method uses three normalization techniques simultaneously. In first paper about the MACONT method (Wen et al, 2020) authors propose the method by using three linear normalization techniques: sum-based normalization, ratio-based normalization, and max-min normalization. Then a newest paper is written by Nguyen (2023) have made analysis by different combinations of normalization methods. MACONT T6 Model contains ratio based linear normalization, linear max-min normalization, and vector normalization techniques. Since the other MCDM methods used (DNMARCOS and AROMAN) also these normalizations, T6 model is considered in this paper.

MACONT-T6 has 6 calculation steps (Nguyen, 2023: 10490-10491; Wen et al., 2020: 862-863):

Step 1: Establish a decision matrix same as in other methods used in this paper.

Step 2: Establish three different normalization matrices by using linear ratio based, linear max-min, and vector normalization techniques shown in range of Equation 3-8.

Step 3: Compute the normalized balance values ($\hat{x}_{ij}$).

$$\hat{x}_{ij} = p.\hat{x}'_{ij} + q.\hat{x}''_{ij} + (1-p-q)\hat{x}'''_{ij} \quad (54)$$

Where the balance parameters get reel values between 0 and 1 ($0 \le p, q \le 1$) and determined by decision makers and/or experts.

Step 4: Calculate the $S_{1(a_i)}$ and $S_{2(a_i)}$ values as follows:

$$S_{1(a_i)} = \delta \frac{\rho_i}{\sqrt{\sum_{i=1}^{m}(\rho_i)^2}} + (1-\delta)\frac{\varsigma_i}{\sqrt{\sum_{i=1}^{m}(\varsigma_i)^2}} \quad (55)$$

$$S_{2(a_i)} = \vartheta.\max\left(w_j.\left(\hat{x}_{ij} - \bar{x}_j\right)\right) + (1-\vartheta).\min\left(w_j.\left(\hat{x}_{ij} - \bar{x}_j\right)\right) \quad (56)$$

$$\rho_i = \sum_{j=1}^{n} w_j.\left(\hat{x}_{ij} - \bar{x}_{ij}\right), \quad i = 1,2,\ldots,m \quad (57)$$

$$\varsigma_i = \frac{\prod_{\gamma=1}^{n}(\bar{x}_j - \hat{x}_{ij})^{w_j}}{\prod_{\eta=1}^{n}(\hat{x}_{ij} - \bar{x}_j)^{w_j}} \quad (58)$$

Where w_j denotes the weights of the criteria, γ denotes the criteria set that satisfy the $\hat{x}_{ij} < \bar{x}_j$, and η acts the criteria meet the condition of $\hat{x}_{ij} \ge \bar{x}_j$. Furthermore δ and ϑ are preference parameters for comprehensive performances and best performances of alternatives taking the value between zero and one. ρ_i is the arithmetic weighted aggregation operator while ς_i is geometric weighted aggregation operator.

Step 5: Compute the final comprehensive score $S(a_i)$ for each alternative.

$$S(a_i) = \frac{1}{2}\, S_{1(a_i)} + \frac{S_{2(a_i)}}{\sqrt{\sum_{i=1}^{m}\left(S_{1(a_i)}\right)^2}}, \quad i = 1,2,\ldots,m. \quad (59)$$

Step 6: Rank the alternatives using final comprehensive scores. The highest score indicates best alternative.

2.6. CRADIS Method

The CRADIS method, which is a combination of the application steps of ARAS, MARCOS and TOPSIS methods, is a multi-criteria decision-making method based on the deviation of alternatives from ideal and anti-ideal solutions. The CRADIS method uses a ratio-based linear normalization technique. In the method, the alternatives are ranked according to the average deviations from the utility degrees obtained at the end of the process steps (Puska et al., 2022).

2.7. MAUT Method

The MAUT method, which is based on the multi attribute utility theory proposed by Keeney and Raiffa in 1976, argues that there is a utility value or function to be maximized that reflects the criteria to be used in evaluating alternatives in a decision problem (Nikou, 2011; Zietsman et al., 2006). The utility values of alternatives for each criterion are determined by max-min linear normalization process (Aytaç Adalı & Tuş Işık, 2017).

2.8. MOOSRA Method

The MOOSRA method, which is one of the multi-objectives, multi-criteria decision-making methods, is often referred to in the literature together with another multi-criteria decision-making method, the MOORA method. The MOOSRA method, which is like the MOORA ratio approach, differs from the MOORA method in the process step where performance scores are obtained. In the MOORA method, the performance scores of the alternatives are calculated by the difference between the weighted normalized values of the benefit and cost criteria, while in the MOOSRA method, this score is obtained by proportioning (Ulutaş, 2020). Thanks to this proportioning method, negative performance scores are prevented (Jagadish & Ray, 2014). In the method using vector normalization, the alternative with the highest performance score is selected as the best alternative.

3. Results

In this section of the study, firstly, the criteria weights are determined by MPSI method. The findings of MPSI weighted DNMARCOS, AROMAN and MACONT-T6 analyses are shared. Comparative analyses of these methods using multiple normalization with each other and with the results of methods using a single normalization function (CRADIS, MOOSRA, MAUT) are also included at the end of the section.

3.1. Determining Criteria Weights Using MPSI Method

The criteria weights in Table 3 were obtained by following the MPSI method steps in the range of Equation 9-13 with the data set in Appendix 1 as the initial decision matrix. According to the weights obtained, the price of the product (C1) was found to be the most important criterion, followed by suction (C2) and wireless working time (C11) features. Among the criteria determined, the criterion with the least effect on the robot vacuum cleaner decision was the product evaluation score (C10).

Table 3. MPSI Weights of Criteria

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
vj	0.5550	0.3838	0.4982	0.5894	0.2630	0.6315	0.4512	0.5903	0.8213	0.9088	0.6840
pj	2.1993	0.9526	0.5055	0.7308	0.7774	0.3451	0.8087	0.4147	0.7677	0.1240	0.9203
wj	0.2574	0.1115	0.0591	0.0855	0.0910	0.0404	0.0946	0.0485	0.0898	0.0145	0.1077

3.2. Selection of Robot Vacuum Cleaner Using DNMARCOS Method

Some parts of the normalized decision matrices obtained by Equations 15 to 20 in the DNMARCOS method where the data set in Appendix 1 is used as the initial decision matrix are given in Table 4 and Table 5.

Table 4. Extended Linear Ratio-Based Normalization Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	0.1891	0.6296	0.5000	0.5000	0.6923	1.0000	0.5000	0.6667	0.2229	0.8043	0.4412
A2	0.2117	0.8500	0.5000	0.8000	0.7692	0.8125	0.4000	0.7450	0.2943	0.8409	0.6818
A3	0.2758	0.6296	0.5000	0.5000	0.7692	0.8125	1.0000	0.7167	0.2700	0.7551	0.5000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A40	0.2159	0.7727	0.5000	0.4444	0.4615	0.9000	0.5000	0.5167	0.2286	0.9024	0.4167
A41	0.2507	0.6800	0.5000	0.3333	0.9231	0.8000	0.5000	0.6167	0.2286	0.7872	0.5000
$a_j'^{(-)}$	*1.0000*	*0.2125*	*0.2500*	*0.2667*	*1.0000*	*1.0000*	*0.2500*	*1.0000*	*1.0000*	*0.7400*	*0.3409*
$a_j'^{+)}$	*0.1755*	*1.0000*	*1.0000*	*1.0000*	*0.1538*	*0.5000*	*1.0000*	*0.3667*	*0.2229*	*1.0000*	*1.0000*

Table 5. Extended Vector Normalization Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	0.9356	0.8724	0.8470	0.8622	0.8461	0.8075	0.8348	0.8388	0.8887	0.8422	0.8277
A2	0.9279	0.9055	0.8470	0.9139	0.8290	0.8436	0.7935	0.8199	0.8531	0.8490	0.8885
A3	0.9061	0.8724	0.8470	0.8622	0.8290	0.8436	0.9174	0.8267	0.8652	0.8319	0.8479
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A40	0.9265	0.8960	0.8470	0.8450	0.8974	0.8268	0.8348	0.8751	0.8859	0.8593	0.8175
A41	0.9147	0.8818	0.8470	0.7933	0.7949	0.8460	0.8348	0.8509	0.8859	0.8388	0.8479
$a_j'^{(-)}$	*0.9403*	*0.6219*	*0.6940*	*0.7416*	*0.9658*	*0.9038*	*0.6695*	*0.9113*	*0.8887*	*0.8285*	*0.7770*
$a_j'^{+)}$	*0.6596*	*0.9196*	*0.9235*	*0.9311*	*0.7778*	*0.8075*	*0.9174*	*0.7582*	*0.5008*	*0.8731*	*0.9240*

The subordinate values obtained by Equations 27-35 are given in Table 6, the utility degrees of the alternatives are given in Table 7, and finally the utility values and DNMARCOS rankings of the alternatives are given in Table 8.

Table 6. Subordinate Values of Alternatives

Alternatives	Si (1)	Si(2)	Si(3)	Alternatives	Si (1)	Si(2)	Si(3)
A1	0.4534	0.2087	0.8713	⋮	⋮	⋮	⋮
A2	0.5361	0.2029	0.8757	**A37**	0.4523	0.1897	0.8749
A3	0.5347	0.1864	0.8712	**A38**	0.5362	0.1663	0.8662
A4	0.4990	0.1578	0.8518	**A39**	0.4937	0.1842	0.8488
A5	0.4806	0.2036	0.8741	**A40**	0.4385	0.2018	0.8764
A6	0.4915	0.1905	0.8634	**A41**	0.4777	0.1928	0.8604
A7	0.4920	0.2120	0.8732	***AID***	*0.6227*	*0.2574*	*0.8361*
⋮	⋮	⋮	⋮	***ID***	*0.4824*	*0.0878*	*0.7832*

Table 7. Utility Degrees of Alternatives

	CCM		UCM		ICM	
Alternatives	**hi (1) (-)**	**hi (1) (+)**	**hi (2) (-)**	**hi (2) (+)**	**hi (3) (-)**	**hi (3) (+)**
A1	0.7281	0.9399	0.8109	2.3773	1.0421	1.1126
A2	0.8608	1.1112	0.7883	2.3111	1.0474	1.1182
A3	0.8587	1.1084	0.7242	2.1233	1.0420	1.1125
A4	0.8013	1.0343	0.6131	1.7974	1.0188	1.0877
A5	0.7718	0.9963	0.7911	2.3193	1.0454	1.1161
⋮	⋮	⋮	⋮	⋮	⋮	⋮
A36	0.7013	0.9053	0.7607	2.2304	1.0393	1.1096
A37	0.7263	0.9375	0.7370	2.1609	1.0464	1.1171
A38	0.8610	1.1114	0.6463	1.8947	1.0360	1.1061
A39	0.7928	1.0233	0.7159	2.0989	1.0152	1.0838
A40	0.7041	0.9089	0.7841	2.2990	1.0482	1.1190
A41	0.7671	0.9902	0.7493	2.1969	1.0291	1.0986

Table 8. Utility Values of Alternatives

Alternatives	CCM		UCM		ICM		λi	Rank
	λi (1) (-)	λi (1) (+)	λi (2) (-)	λi (2) (+)	λi (3) (-)	λi (3) (+)		
A1	0.5635	0.4365	0.7457	0.2543	0.5163	0.4837	0.6696	8
A2	0.5635	0.4365	0.7457	0.2543	0.5163	0.4837	0.6967	1
A3	0.5635	0.4365	0.7457	0.2543	0.5163	0.4837	0.6755	4
A4	0.5635	0.4365	0.7457	0.2543	0.5163	0.4837	0.6222	29
A5	0.5635	0.4365	0.7457	0.2543	0.5163	0.4837	0.6752	5
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A36	0.4365	0.7457	0.2543	0.5163	0.4837	0.4365	0.6472	21
A37	0.4365	0.7457	0.2543	0.5163	0.4837	0.4365	0.6477	19
A38	0.4365	0.7457	0.2543	0.5163	0.4837	0.4365	0.6510	16
A39	0.4365	0.7457	0.2543	0.5163	0.4837	0.4365	0.6504	17
A40	0.4365	0.7457	0.2543	0.5163	0.4837	0.4365	0.6570	14
A41	0.4365	0.7457	0.2543	0.5163	0.4837	0.4365	0.6575	13

As a result of the MPSI-weighted DNMARCOS analysis, the three best alternatives were A2. A7 and A32. while A25, A23 and A26 were ranked last.

3.3. Selection of Robot Vacuum Cleaner Using AROMAN Method

A part of the linear max-min normalization matrix obtained by Equation 5-6 in AROMAN method is given in Table 9. On the other hand, since the second normalization method of the method, vector normalization, is also used in DNMARCOS method, the same matrix is not given here again. The vector normalization matrix of the AROMAN method also consists of the normalized values in Table 5. The matrix containing the normalized values integrated with the help of Equation 49 is shown in Table 10 and the MPSI weighted aggregated normalized matrix is shown in Table 11.

Table 9. Linear Max-Min Normalization Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	0.0166	0.1587	0.3333	0.3636	0.6364	1.0000	0.3333	0.4737	0.0000	0.6923	0.6552
A2	0.0439	0.0476	0.3333	0.0909	0.7273	0.6250	0.5000	0.5974	0.0919	0.5385	0.2414
A3	0.1216	0.1587	0.3333	0.3636	0.7273	0.6250	0.0000	0.5526	0.0607	0.9231	0.5172
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A40	0.0490	0.0794	0.3333	0.4545	0.3636	0.8000	0.3333	0.2368	0.0074	0.3077	0.7241
A41	0.0912	0.1270	0.3333	0.7273	0.9091	0.6000	0.3333	0.3947	0.0074	0.7692	0.5172

Table 10. Aggregated Normalization Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	0.2380	0.2578	0.2951	0.3065	0.3706	0.4519	0.2920	0.3281	0.2222	0.3836	0.3707
A2	0.2430	0.2383	0.2951	0.2512	0.3891	0.3671	0.3234	0.3543	0.2362	0.3469	0.2825
A3	0.2569	0.2578	0.2951	0.3065	0.3891	0.3671	0.2293	0.3448	0.2315	0.4387	0.3413
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A40	0.2439	0.2438	0.2951	0.3249	0.3153	0.4067	0.2920	0.2780	0.2233	0.2918	0.3854
A41	0.2515	0.2522	0.2951	0.3801	0.4260	0.3615	0.2920	0.3114	0.2233	0.4020	0.3413

Table 10. Weighted Aggregated Normalization Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	0.0613	0.0287	0.0175	0.0262	0.0337	0.0182	0.0276	0.0159	0.0200	0.0056	0.0399
A2	0.0625	0.0266	0.0175	0.0215	0.0354	0.0148	0.0306	0.0172	0.0212	0.0050	0.0304
A3	0.0661	0.0287	0.0175	0.0262	0.0354	0.0148	0.0217	0.0167	0.0208	0.0064	0.0368
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A40	0.0628	0.0272	0.0175	0.0278	0.0287	0.0164	0.0276	0.0135	0.0201	0.0042	0.0415
A41	0.0647	0.0281	0.0175	0.0325	0.0388	0.0146	0.0276	0.0151	0.0201	0.0058	0.0368

Table 11. Separately Normalized Weighted Values and Rankings of Alternatives

	Li	Ai	Ri	Rank
A1	0.1491	0.1455	0.7676	25
A2	0.1512	0.1315	0.7515	38
A3	0.1539	0.1372	0.7627	28
A4	0.1582	0.1458	0.7796	14
A5	0.1448	0.1419	0.7573	35
⋮	⋮	⋮	⋮	⋮
A36	0.1457	0.1498	0.7688	24
A37	0.1420	0.1388	0.7493	39
A38	0.1538	0.1359	0.7608	30
A39	0.1608	0.1464	0.7835	12
A40	0.1414	0.1458	0.7579	34
A41	0.1532	0.1483	0.7765	16

The final AROMAN ranking scores obtained with the help of Equations 51. 52. and 53 are given in Table 11. The most successful alternatives of the AROMAN method, which uses a combination of max-min linear and vector normalization techniques, founded A10. A14 and A19, respectively.

3.4. Selection of Robot Vacuum Cleaner Using MACONT-T6 Method

The MACOT-t6 method uses the Linear Ratio Base 0-1 Interval normalization method in common with the DNMARCOS method, and the Iinear Max-Min normalization method in common with the AROMAN method. Therefore, the normalized matrices for the method are the same as the previously calculated Tables 3 and 8. The normalized matrix balanced with the help of Equation 54 is given in Table 12.

Tablo 12. Normalized Balanced Values of MACONT-T6

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	0.9490	0.4562	0.5601	0.5864	0.4773	0.4358	0.5560	0.6384	0.9629	0.8182	0.7519
A2	0.9043	0.4010	0.5601	0.4460	0.4339	0.6113	0.6395	0.5716	0.8395	0.7558	0.5433
A3	0.8070	0.4562	0.5601	0.5864	0.4339	0.6113	0.3891	0.5952	0.8766	0.9117	0.6823
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A40	0.8969	0.4168	0.5601	0.6332	0.6224	0.5274	0.5560	0.7826	0.9512	0.6623	0.7866
A41	0.8412	0.4404	0.5601	0.7735	0.3508	0.6237	0.5560	0.6836	0.9512	0.8493	0.6823

The values of ρ_i, ς_i, $S_{1(a_i)}$ and $S_{2(a_i)}$ obtained in the fourth stage of the MACONT method and the final comprehensive scores $S(a_i)$ of the alternatives are presented in Table 13. The final ranking scores indicate that A40 is the best alternative, followed by A1 in second place.

Tablo 13. Final Comprehensive Scores $S(a_i)$ and Rankings of Alternatives

	ρ_i	ς_i	$S_{1(a_i)}$	$S_{2(a_i)}$	$S_{(a_i)}$	Rank
A1	0.1145	2.0327	0.1817	0.0263	0.1943	2
A2	0.0056	0.2854	0.0112	0.0166	0.0710	16
A3	-0.0032	0.2764	-0.0010	0.0054	0.0208	25
A4	-0.0059	0.0614	-0.0073	0.0020	0.0043	28
A5	0.0464	0.7477	0.0727	0.0202	0.1161	10
⋮	⋮	⋮	⋮	⋮	⋮	⋮
A36	0.0639	7.7988	0.1836	0.0146	0.1493	9
A37	0.0359	5.2650	0.1140	0.0136	0.1107	11
A38	-0.0161	12.6237	0.1337	-0.0002	0.0662	18
A39	0.0054	3.1171	0.0458	0.0018	0.0298	24
A40	0.0725	25.8775	0.4181	0.0195	0.2858	1
A41	0.0386	5.6811	0.1229	0.0094	0.0983	13

4. Comparative Analysis

Comparative analyzes were carried out with different approaches based on the literature in this section. There are many research in the literature comparing different multi-criteria decision-making methods. Some of these studies focus on the relationship between the final scores of the methods or their similarities/differences (Abdulaal & Bafail, 2022; Baydaş & Pamucar, 2022; Kizielewicz et al., 2023). These papers utilize the correlations between the rankings produced by the methods to evaluate the relationship and similarity and the standard deviations of the ranking scores to compare the methods in terms of the variability of the results. Comparative evaluations of the methods used based on the correlation analysis between the final ranking scores and the examination of the variability of these scores are given in section 4.1.

Some research in the literature compares different MCDM methods based on sensitivity analysis (Lee & Chang, 2018; Mulliner et al., 2016). The responses of the alternative rankings produced by the methods to the criteria weights are evaluated in this type of study. Sensitivity analyses were also made regarding the weights of the evaluation criteria considered in this research. The sensitivities of both the criteria and the MCDM methods used were evaluated against changing weights in section 4.2.

In recent years, a number of papers benchmark different MCDM methods in terms of the algorithm structures they use (Alkahtani, 2019; Ghaleb, 2020; Junior et al., 2014). In this group, Junior et al. (2014)'s study compares multi-criteria decision-making methods under four titles: (1) Adequacy to changes of alternatives, (2) adequacy to changes of criteria, (3) agility in the decision process, and (4) computational complexity. Junior et al. (2014), compare the adequacy/appropriateness of alternatives and criteria titles in relation to sensitivity analyzes regarding whether the final rankings differ in case a new alternative or criterion is added or removed from the problem and evaluation of the problem according to the number of alternatives and criteria required for the healthy application of MCDM methods. In their study, the authors compared fuzzy AHP and TOPSIS methods on a small decision problem with five alternatives and five criteria. They found that even on this small problem, there was no statistically significant difference in the final rankings. Moreover, other studies used as references in this paper (e.g., Alkahtani, 2019; Ghaleb, 2020) have not also analyzed the adding or removing alternatives/criteria to the decision problem. On the other hand, these two comparison criteria are thought to be more suitable for methods such as AHP and ANP, where pairwise comparisons are made by decision makers and the consistency of pairwise comparisons is mandatory. Since there are many alternatives and criteria in the decision problem under consideration, the criterion or alternative to be added will most likely not create a statistically significant difference in the final rankings; due to the lack of a rule base regarding the number of alternatives and/or criteria for the application of the MCDM methods discussed in the study, these two comparison criteria proposed by Junior et al. (2014) are not used in this research.

"Agility in the decision process", another method comparison criterion proposed by Junior et al. (2014), is related to the number of judgments required from the decision maker in the methods used, in other words, the size of the initial decision matrix used by the methods. Since all the methods considered and compared in the study use the same initial decision matrix, the methods are not compared according to the agility of the decision process.

Since only the computational complexity of the method comparison steps suggested by Junior et al. (2014) is appropriate for this study, the methods are compared in terms of their complexity. Computational complexity, a criterion related to the number of operations involved in a multi-criteria decision-making method, is discussed in detail under section 4.3.

4.1. MCDM Methods’ Benchmarks

Correlation analysis (Bandyopadhyay, 2020; Mathew & Sahu, 2018) and standard deviation approach (Baydaş & Pamucar, 2022; Salabun & Piegat, 2017; Zaidan et al., 2017) are frequently used methods for comparing the performance of methods in the multi-criteria decision-making literature. The similarity and differences in the performance of methods can be evaluated by correlation analysis of scores or ranking results obtained from different methods.

In the final stages of the study, all the results obtained, and the scores used to rank the alternatives are listed in Table 13. The correlations between the scores are shown in the Correlation Heat Map in Figure 1. The results of the correlation analysis can be evaluated in three categories: 1) Relationships between methods using multiple normalizations 2) Relationships between methods using multiple normalizations and methods using a single normalization function 3) Relationships between methods using a single normalization function. Accordingly, the two methods with the highest relationship between scores in the first category are DNMARCOS and AROMAN methods. A strong negative relationship (r=-0.8) is found between the ranking scores of these two methods. In the second category, a very high positive relationship (r=0.93) is observed between MACONT-T6 and MAUT methods, as well as between MACONT-T6 and CRADIS, while a significant relationship is found between the scores of MACONT-T6 and MOOSRA. In the third and final category, the methods producing the highest similarity results are MAUT and CRADIS (r=0.91).

Considering the correlation coefficients between the final scores of the MCDM methods, it can be said that the normalization techniques used by the DNMARCOS method (linear ratio-based normalization, vector normalization) do not exhibit dominance over each other, and their effects on the final ranking are similar. On the other hand, it is seen that the vector normalization technique in AROMAN method and the linear normalization techniques in MACONT-T6 method have a more dominant effect on the final ranking of the alternatives than the other normalization techniques they use.

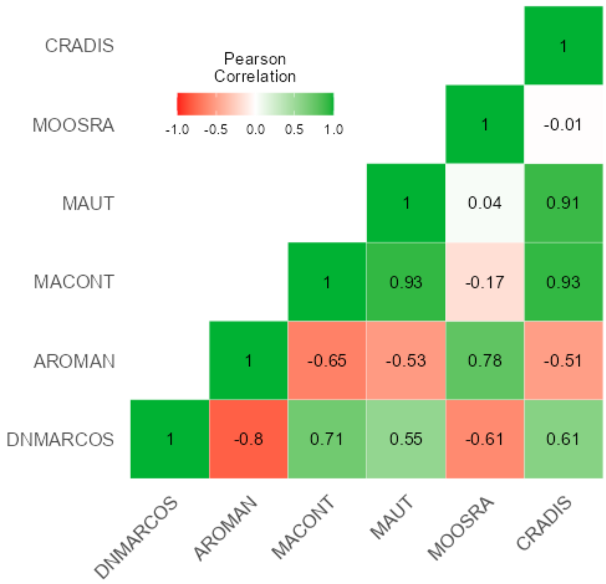
Table 14. Final Scores and Rankings of Alternatives of the MCDM Methods

	DNMARCOS		AROMAN		MACONT		CRADIS		MOOSRA		MAUT	
	Score	**Rank**	**Score**	**Rank**	**Score**	**Rank**	**Score**	**Rank**	**Score**	**Rank**	**Score**	**Rank**
A1	0.670	8	0.768	25	0.194	2	0.996	2	0.152	25	0.582	10
A2	0.697	1	0.751	38	0.071	16	0.890	20	0.131	39	0.501	29
A3	0.676	4	0.763	28	0.021	25	0.855	23	0.134	37	0.510	28
A4	0.622	29	0.780	14	0.004	28	0.845	28	0.162	17	0.531	21
A5	0.675	5	0.757	35	0.116	10	0.943	10	0.143	32	0.579	12
A6	0.661	10	0.770	22	0.049	21	0.890	19	0.150	26	0.532	20
A7	0.689	2	0.759	32	0.159	7	0.993	3	0.145	30	0.553	18
A8	0.638	25	0.772	21	0.007	27	0.849	25	0.152	24	0.527	23
A9	0.674	6	0.767	26	0.180	4	1.000	1	0.153	23	0.585	6
A10	0.639	24	0.788	10	-0.010	31	0.816	32	0.154	21	0.488	32
A11	0.629	27	0.773	20	0.106	12	0.892	18	0.168	12	0.576	13
A12	0.620	31	0.769	23	-0.273	41	0.685	41	0.109	41	0.262	41
A13	0.620	30	0.759	33	-0.099	34	0.754	38	0.126	40	0.427	35
A14	0.641	23	0.756	36	0.051	20	0.917	13	0.153	22	0.602	5
A15	0.609	32	0.795	7	-0.010	30	0.847	26	0.184	5	0.520	26
A16	0.630	26	0.792	8	0.070	17	0.896	17	0.179	6	0.570	15
A17	0.623	28	0.791	9	0.008	26	0.868	22	0.175	7	0.541	19
A18	0.561	37	0.829	1	-0.180	37	0.749	39	0.210	1	0.354	40
A19	0.659	12	0.787	11	0.079	14	0.900	15	0.165	14	0.564	16
A20	0.584	35	0.799	6	-0.146	35	0.742	40	0.164	15	0.398	36
A21	0.558	38	0.825	2	-0.222	40	0.796	35	0.203	3	0.383	38
A22	0.648	20	0.776	17	0.032	23	0.843	30	0.149	28	0.510	27
A23	0.550	40	0.783	13	0.052	19	0.900	16	0.169	10	0.555	17
A24	0.564	36	0.803	5	-0.207	39	0.765	37	0.159	19	0.380	39
A25	0.551	39	0.809	4	-0.206	38	0.778	36	0.169	11	0.392	37
A26	0.549	41	0.815	3	-0.152	36	0.806	34	0.196	4	0.428	34
A27	0.649	18	0.778	15	0.183	3	0.969	6	0.171	8	0.624	2
A28	0.669	9	0.764	27	0.072	15	0.939	12	0.166	13	0.583	9
A29	0.661	11	0.756	37	0.004	29	0.843	29	0.133	38	0.500	31
A30	0.645	22	0.761	31	-0.022	32	0.814	33	0.135	36	0.479	33
A31	0.589	34	0.773	19	-0.034	33	0.845	27	0.162	16	0.524	25
A32	0.676	3	0.746	41	0.153	8	0.969	5	0.135	35	0.585	7

Table 14. Final Scores and Rankings of Alternatives of the MCDM Methods (Continue)

	DNMARCOS		AROMAN		MACONT		CRADIS		MOOSRA		MAUT	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
A33	0.603	33	0.775	18	0.165	6	0.958	9	0.204	2	0.672	1
A34	0.671	7	0.746	40	0.167	5	0.963	8	0.138	33	0.584	8
A35	0.657	15	0.762	29	0.049	22	0.851	24	0.145	29	0.525	24
A36	0.647	21	0.769	24	0.149	9	0.966	7	0.158	20	0.617	4
A37	0.648	19	0.749	39	0.111	11	0.905	14	0.145	31	0.580	11
A38	0.651	16	0.761	30	0.066	18	0.828	31	0.136	34	0.500	30
A39	0.650	17	0.784	12	0.030	24	0.889	21	0.170	9	0.528	22
A40	0.657	14	0.758	34	0.286	1	0.985	4	0.150	27	0.618	3
A41	0.658	13	0.777	16	0.098	13	0.940	11	0.160	18	0.571	14
Std.Dev	*0.273*		*0.243*		*0.227*		*0.204*		*0.212*		*0.247*	

Figure 1. Correlation Heat Map for the Methods' Final Scores



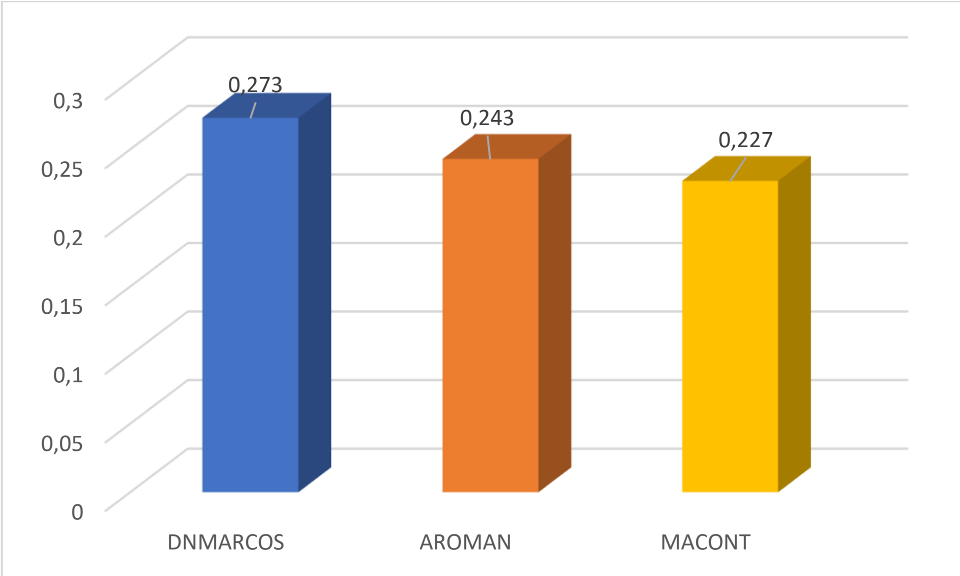
Another approach used in comparing the results of MCDM methods is the standard deviation approach (Baydaş & Pamucar, 2022; Zaidan et al., 2017). Baydaş and Pamucar (2022) proposed the two-stage standard deviation method, based on the study of Wang and Rangaiah (2017). In this method, first the final scores obtained because of different methods are normalized with the max-min linear normalization technique (Equation 60). Then, the variations are evaluated by calculating the standard deviations of the normalized scores (Equation 61).

$$F_{ij} = \frac{f_{ij} - minf_{ij}}{maxf_{ij} - minf_{ij}} \tag{60}$$

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m}(F_{ij} - \bar{F}_{ij})^2}{m}} \qquad j = 1.2. \dots , n \tag{61}$$

The bar chart of the calculated standard deviation scores of the methods used in the study is shown in Figure 2. Accordingly, among the methods that use multiple normalization functions, the method with the highest standard deviation was DNMARCOS, while the method that produced a score with the lowest standard deviation was the MACONT-T6 method. The MAUT method provided ranking with scores with the lowest variability among all methods. While low variability is desired in many statistical interpretations, this situation is interpreted in the opposite way in method comparison. It can be concluded that the method that produces a ranking score with a high standard deviation achieves a ranking by separating the alternatives more clearly. In this case, it can be said that the performance of DNMARCOS is better than other methods among the methods that use multiple normalization methods.

Figure 2. Standard Deviations of Methods



4.2. Sensitivity Analysis

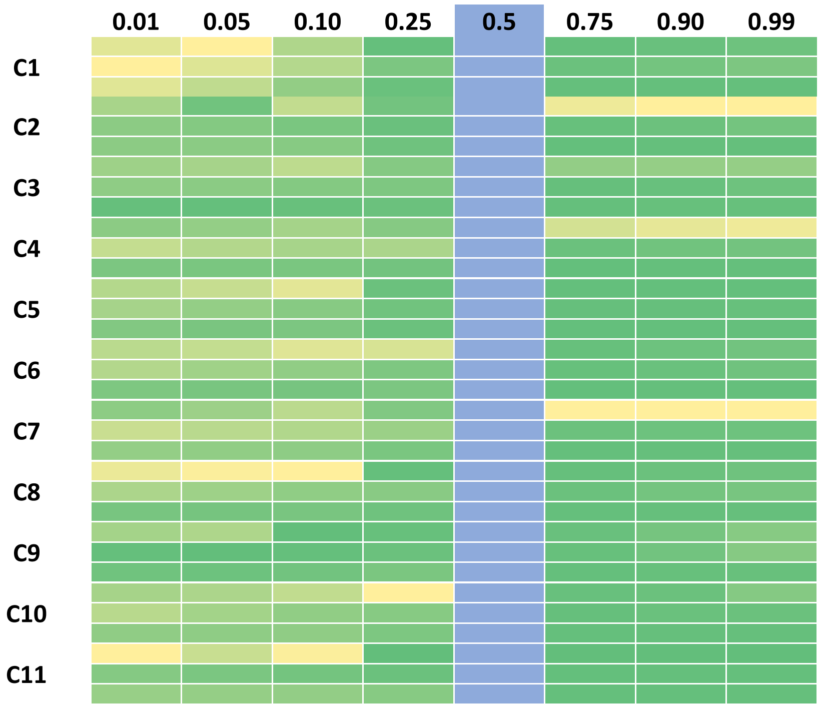
In MCDM problems, criterion weights have a significant impact on the final rankings. Criteria weights may affect the final scores of different ranking methods differently. For this reason, it is also important to perform sensitivity analyses regarding criterion weights in method comparison studies.

Weighting scenarios were created in order to make comparisons between MCDM methods used. Each criterion was weighted with 0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 0.90 and 0.99 in the scenarios, respectively, the other criteria were assumed to be equally weighted. While determining the weight coefficients, it was ensured that the sum of the criteria weights in the scenario was 1.00. 88 scenarios were analyzed for each of DNMARCOS, AROMAN and MACONT-T6 methods and 264 scenarios in total. Figure 3 was prepared with the Spearman rank correlation coefficients between the rankings obtained from the scenarios.

In order to compare the results obtained from DNMARCOS, AROMAN and MACONT-T6 methods more clearly, coloring was done according to the magnitude of the correlation coefficients based on scenarios where the weight of each criterion is 0.5. Accordingly, dark green cells mean that the alternative ranking obtained when the criterion weight takes the value at the beginning of the column does not change much. When the color goes towards yellow, it means that the correlation between the rankings decreases. For each criterion, the first row represents the correlation analysis results obtained from DNMARCOS, the second row from AROMAN and the third row from MACONT-T6 method.

According to the results, the criterion with the least sensitivity to the criteria weights is C9 (Height), followed by C5 (Charging Time) and C11 (Wireless Working Time). The most sensitive criteria to the weights are C4 (Hopper Capacity), C2 (Suction) and C7 (Number of Cleaning Modes). When the sensitivity is evaluated in terms of methods, it is seen that the rankings of the DNMARCOS method are highly affected by the changes in the criteria weights. MACONT-T6 method is the least affected by the weights and had the lowest sensitivity.

Figure 3. The Sensitivity Analysis Results with Changed Weights



4.3. Complexity of MCDM Methods

Computational complexity is a measure used to compare multi-criteria decision-making methods. All suggested approaches' computational complexity was assessed by examining time complexity, T, within the calculations, considering the number of augmentations (Alkahtani et al., 2019; Chang, 1996; Ghaleb et al., 2020; Junior et al., 2014). The number of operations used by the methods used in the study until the final ranking is obtained, where n is the number of alternatives and m is the number of criteria, and the complexity levels can be calculated as follows:

- DNMARCOS method has 2nm operations in normalization steps, 4m operations in calculating ELNM and EVLNM values, 2m(n+2) for calculating weighted ELNM and EVLNM values, 3(n+2) for calculating secondary values, 6n for utility degrees, 6n for utility values, and 6n operations for general utility values. The computational complexity value for the DNMARCOS method is calculated with the Equation 62

$$T_{DNMARCOS} = 4mn + 8m + 10n + 6 \quad (62)$$

- In the AROMAN method, the complexity score, including 2(mn) operations in the normalization stage, mn for the aggregated normalization matrix, mn for the weighted normalized matrix, n for the sum of the weighted normalized values, and n for the final ranking scores of the alternatives, is calculated with Equation 63.

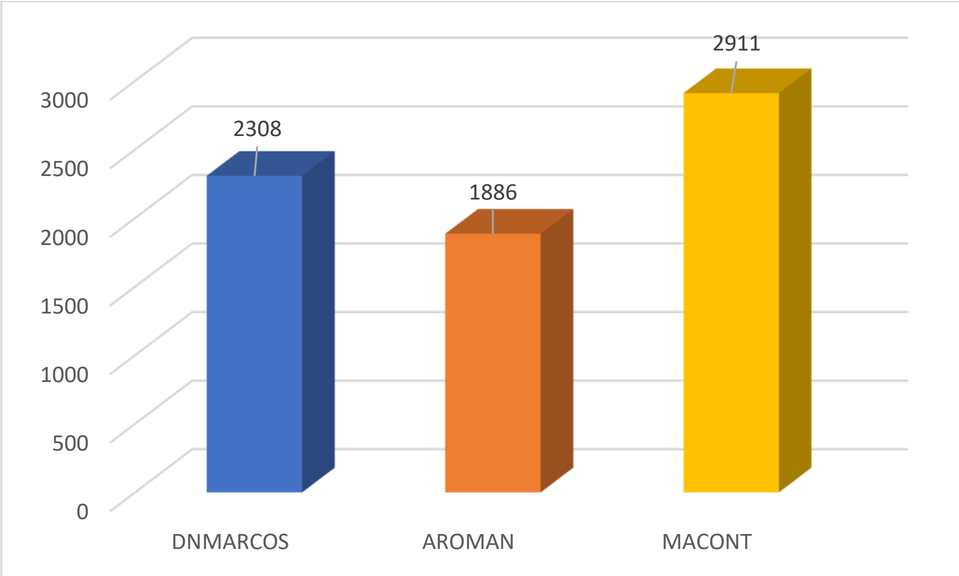
$$T_{AROMAN} = 4mn + 2n \quad (63)$$

- The complexity value of the MACONT-T6 method includes 3mn operations from normalization steps, mn from balanced normalized values, 2mn+5n operations from the remaining steps and is calculated with Equation 64.

$$T_{MACONT} = 6mn + 5n \quad (64)$$

The results of the methods regarding the computational complexity are shown in the bar chart in Figure 3. Among the methods using multiple normalization, AROMAN was the method with the lowest complexity, while the MACONT-T6 method was the method with the highest computational complexity.

Figure 3. Computational Complexity of Methods



5. Conclusion

There are lots of proposed multi-criteria decision-making method for solving complex decision problems that encompass conflicting multiple evaluation criteria and numerous alternatives. One of the most important distinguishing features of these methods is their normalization techniques. Using different normalization functions that standardize evaluation criteria units allows for different final rankings in multi-criteria decision-making. To produce robust results, there has been an increase in the use of multi-criteria decision-making methods that combine more than one normalization technique. This study presents a comparative evaluation by applying the DNMARCOS, AROMAN, and MACONT methods to the same decision problem under the same conditions. The comparative analysis is divided into three groups: 1) Comparison of the results of methods applied with multiple normalization techniques (DNMARCOS, AROMAN, and MACONT) and methods applied with single normalization techniques (MAUT, MOOSRA, CRADIS) based on correlations 2) Comparison of the results of methods applied with multiple normalization techniques based on variabilities in final ranking scores 3) Comparison of the results of DNMARCOS, AROMAN, and MACONT based on sensitivity analysis 4) Comparison of the performances of algorithmic structures of DNMARCOS, AROMAN, and MACONT methods based on computational complexity.

The results of the comparative analysis in the first group showed that DNMARCOS and AROMAN have a high-level correlation in the opposite direction. The opposite direction in the ranking is the same-directional normalization expansion features for the benefit and cost criteria in AROMAN, and the criterion with a clear lead weight is minimization-oriented. Despite using two common normalization functions, the relationship between the final scores of DNMARCOS and MACONT methods is at a lower level.

Notably, the scores of MAUT, MOOSRA, and CRADIS methods using different normalization methods are highly similar, while the results of these methods and the results of multiple normalization methods are at a moderate level. This situation arises from the diversity of aggregation methods with multiple normalizations. The diversity and differences in ranking scores of different weight methods used in the aggregation function will increase and decrease.

The sensitivity analysis results show that AROMAN and MACONT have more robust performances, while DNMARCOS rankings are more sensitive than the other methods in higher criteria weights. As the criterion weights decrease, serious changes are observed in the rankings. When a general evaluation was made, it was noted that the AROMAN method showed a better performance in terms of sensitivity.

The last comparison system evaluated the performances of DNMARCOS, AROMAN, and MACONT methods regarding programming complexity and standard deviations of final scores. According to both performance criteria, the worst-performing method was the MACONT method. While the DNMARCOS method, which produces ranking scores with high standard deviation, had the most minor calculation steps and the lowest complexity, the method with the least calculating steps and the lowest complexity was AROMAN.

Considering all the comparison results, it can be said that there is a trade-off between the advantages and disadvantages of the multi-criteria decision-making methods. A method with the best performance according to one benchmark may perform poorly according to another criterion. At this point, the decision maker's definition of performance will be essential and distinctive in performance comparison criterion evaluation. Aggregating the results of all benchmarks with an analytical method can also be suggested as another approach for the decision-maker.

Different normalization techniques can be used together and separately to evaluate the effects of different multi-criteria decision-making techniques on the results in future studies. Papers can be conducted on the performance of methods using multiple normalization in fuzzy and/or gray decision environments involving incomplete information and uncertainty.

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Appendix

Appendix 1. Data Set

	min	max	max	max	min	min	max	min	min	max	max
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	6790.00	2700.00	20.00	400.00	4.50	80.00	4.00	4,00	78.00	4.60	170.00
A2	7600.00	2000.00	20.00	250.00	5.00	65.00	5.00	4,47	103.00	4.40	110.00
A3	9900.00	2700.00	20.00	400.00	5.00	65.00	2.00	4,30	94.50	4.90	150.00
A4	13890.00	2700.00	20.00	400.00	4.50	65.00	5.00	4,20	94.50	4.80	150.00
A5	7499.90	2000.00	20.00	600.00	4.00	65.00	3.00	3.30	110.00	4.00	150.00
A6	9325.00	2700.00	20.00	400.00	5.50	65.00	4.00	4,30	94.50	4.40	150.00
A7	6333.00	2000.00	20.00	500.00	5.50	65.00	5.00	4,20	80.00	4.30	110.00
A8	12155.00	2700.00	20.00	400.00	4.50	65.00	4.00	4,30	95.00	4.70	150.00
A9	6300.00	2700.00	20.00	400.00	5.00	65.00	4.00	4,30	95.00	4.70	170.00
A10	14799.00	2000.00	20.00	450.00	6.00	65.00	3.00	4,30	105.00	4.20	200.00
A11	11099.00	4000.00	20.00	450.00	4.00	68.00	4.00	3.56	100.40	4.30	150.00
A12	32999.00	1700.00	20.00	300.00	3.00	69.00	2.00	3.20	87.00	4.60	75.00
A13	19999.00	1700.00	20.00	400.00	3.00	59.00	4.00	3.80	93.00	5.00	75.00
A14	11140.00	3000.00	40.00	385,00	3.00	60.00	2.00	2.20	90.00	4.40	150.00
A15	15648.00	5000.00	20.00	300.00	6.00	65.00	4.00	3.90	100.00	4.40	180.00
A16	11999.00	4000.00	17.00	350.00	6.00	65.00	4.00	3.80	96.00	4.60	220.00
A17	13499.00	4200.00	20.00	470.00	6.00	67.00	3.00	3.60	96.50	4.70	180.00
A18	25999.00	5100.00	20.00	400.00	6.00	67.00	3.00	3.50	350.00	4.60	180.00
A19	10749.00	2700.00	20.00	750.00	6.00	67.00	2.00	3.70	96.50	4.60	180.00
A20	22900.00	3200.00	20.00	300.00	6.00	55.00	3.00	3.90	170.00	4.60	180.00
A21	35899.00	5100.00	20.00	400.00	4.00	68.00	3.00	3.60	96.50	4.60	180.00
A22	12599,00	2000.00	20.00	460.00	6.00	65.00	4.00	3.20	96.50	4.80	150.00
A23	22500.00	2000.00	20.00	480.00	1.00	55.00	5.00	3.50	96.00	4.80	180.00
A24	32000.00	2500.00	20.00	460.00	2.50	67.00	2.00	4,50	96.00	4.70	180.00
A25	32280.00	2500.00	20.00	470.00	2.50	67.00	3.00	4,70	96.00	4.70	180.00
A26	27790.00	5100.00	20.00	400.00	6.00	59.00	3.00	3.50	96.50	4.40	180.00
A27	7999.00	2700.00	20.00	500.00	4.00	75.00	5.00	3.60	105.00	5.00	180.00
A28	8529.00	3400.00	10.00	200.00	4.00	77.00	8.00	3.00	100.50	3.70	150.00
A29	11299.00	2700.00	10.00	400.00	4.00	65.00	4.00	4,50	80.00	4.40	120.00
A30	13799.00	2700.00	10.00	400.00	4.00	65.00	4.00	4,50	80.00	4.40	120.00
A31	17999.90	2700.00	30.00	500.00	3.00	60.00	4.00	3.60	80.00	3.90	120.00
A32	7169.00	2200.00	20.00	600.00	3.00	69.00	3.00	3.30	81.50	4.60	110.00
A33	11998.00	8000.00	20.00	450.00	2.50	45.00	3.00	3.70	102.00	4.60	150.00
A34	7360.00	2700.00	20.00	550.00	4.00	50.00	3.00	3.60	81.00	4.70	110.00
A35	11149.00	4000.00	20.00	450.00	4.60	60.00	2.00	3.80	97.00	4.90	120.00
A36	8589.00	3000.00	20.00	550.00	3.00	80.00	3.00	3.50	82.00	4.60	180.00
A37	9440.00	3000.00	20.00	450.00	3.00	60.00	4.00	3.00	98.00	4.70	110.00
A38	12699.00	2100.00	20.00	500.00	4.00	40.00	3.00	6,00	95.00	4.60	120.00
A39	10199.00	4000.00	20.00	550.00	6.50	72.00	4.00	4,15	98.80	4.60	120.00
A40	7749.00	2200.00	20.00	450.00	3.00	72.00	4.00	3.10	80.00	4.10	180.00
A41	8999.00	2500.00	20.00	600.00	6.00	64,00	4.00	3.70	80.00	4.70	150.00
Max	35899.00	8000.00	40.00	750.00	6.50	80.00	8.00	6,00	350.00	5.00	220.00
Min	6300.00	1700.00	10.00	200.00	1.00	40.00	2.00	2.20	78.00	3.70	75.00