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# The Relation Between Trading Volume and Return Volatility: Evidence from Borsa Istanbul

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**Abstract:** *This study investigates the relationship between volume and volatility in the context of the Mixture of Distribution Hypothesis (MDH) and Sequential Information Arrival Hypothesis (SIAH) with respect to company size in Borsa Istanbul (BIST). Employing the generalized method of moments (GMM) method and granger causality tests, we find statistical evidence supporting the MDH for large-cap stocks, whereas we document no evidence of contemporaneous interaction between volume and volatility for mid-cap and small-cap stocks. This suggests that the dissemination of information in the stock market appears to be primarily through large firms. Our findings for large cap stocks have not changed across economic states. In terms of SIAH, for the stocks of companies of any size, we document uni-directional causality running from volatility to volume but not the other way around which is not consistent with the SIAH. However, we find supporting evidence of the SIAH for large cap stocks during the expansion periods.*

**Keywords:** Mixture of Distribution Hypothesis (MDH), Sequential Information Arrival Hypothesis (SIAH), Trading Volume, Return Volatility, Granger Causality, GMM

**JEL:** G10, G12, G14

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## 1. Introduction

Volume- volatility relation in asset markets has long been a subject of research in financial economics. Two basic approaches namely the Mixture of Distribution Hypothesis (MDH) and Sequential Information Arrival Hypothesis (SIAH) have received notable attention from many academics and industry professionals. According to MDH model, which is introduced by Clark (1973), information flow is considered as a latent common factor which affects both volume and volatility. The model states that the relation between volatility and volume is positive and contemporaneous since information dissemination is contemporaneous. When new information reaches the market all market participants receive the information simultaneously. Hence, the model states that the shift to a new equilibrium is immediate. Therefore, volume and volatility does not possess a lead–lag relation but a contemporaneous correlation.

However, this is contrary to the SIAH, developed by Copeland (1976), which predicts a causal relation between volume and volatility. According to the SIAH, new information arrives into the market in a sequential random fashion. The information signals are not received simultaneously by traders. Hence, the formation of equilibrium is not instantaneous. Responses of different market participants to new information are part of a series of incomplete equilibria. When all market participants receive and react to the information signal, the final equilibrium is reached. Thus, this sequential response to new information is suggested to produce a lead-lag relationship between volatility and volume. To put it another way, lagged values of volatility can be used in forecasting current volume and vice versa.

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This study tests the relationship between volatility and volume in the context of the MDH and SIAH hypotheses in Borsa Istanbul (BIST). The key contributions of this study are two-fold. First, we examine whether volume-volatility relationship differs between small firms and large firms, since no study has documented an evidence about whether the relationship between volatility of return and trading volume differs for companies of different sizes. Some studies from different line of literature (see, e.g., Nofsinger, 2001) find that large firms react differently to macroeconomic information compared to small firms. Investors are found to response rapidly to good news by purchasing large-cap stocks rather than purchasing small-cap stocks. In addition, foreign investors and local institutional investors are generally known to invest in large firms due to their liquidity concerns.[1] Given this, it is very plausible to assume that large firms play a different role in the dissemination of information compared to small firms. Thus, this study, according to our knowledge, is the first that investigates whether volume-volatility relationship differs between small firms and large firms. Second, many studies have documented significant time variation in the conditional volatility of equity returns which means that stocks are much risky assets at some time than others (Hamilton & Lin, 1996). For example, Schwert (1989) investigated several factors that could potentially affect equity volatility and find that the most important determinant of the conditional volatility of equity returns is the level of real economic activity. In a similar vein, Hamilton and Lin (1996) have documented economic recessions account for more than 60% of the volatility of stock returns. Previous studies have not provided any evidence about whether the relation between volume and volatility varies across different states of the business cycle. Thus, it will be useful to see whether the relation between volume and volatility changes across different economic states. We analyze this relation with respect to company size.

Our results suggest that the dissemination of information in the equity market appears to be primarily through large firms. We find statistical evidence that supports the MDH hypothesis only for large-cap stocks, but not for mid-cap and small-cap stocks. Furthermore, our findings have not changed in the sub-periods.

In terms of SIAH, our findings for the stocks of companies of any size show strong evidence of uni-directional causality running from volatility to volume but not the other way around which is not in line with the SIAH. While analyzing the sub-periods, we find supporting evidence for the SIAH only in the expansion period and only for large-cap stocks.

The remainder of our study is organized as follows: Section 2 provides a review of the literature. The data is presented in Section 3. The methodology employed in our study is outlined in Section 4. Section 5 and 6 present descriptive statistics and results, respectively. Section 7 summarizes main conclusion.

## 2. Literature Review

A vast amount of studies investigates the relation between volume and volatility in the context of MDH and SIAH hypotheses. While some of these studies document evidences supporting MDH hypothesis, some of them document evidences supporting SIAH hypothesis. For example, Lamoureux and Lastrapes (1990), employing a sample of 20 actively traded stocks, Anderson (1996), employing five common stocks, and Gallo and Pacini (2000), employing 10 actively traded stocks, provide support for MDH hypothesis in U.S. equity markets. Omran and McKenzie (2000) extend the results of Lamoureux and Lastrapes (1990) to the UK stock market and examines 50 UK stocks and document consistent results with theirs which supports the MDH hypothesis. Bohl and Henke (2003) investigate the validity of the MDH hypothesis with 20 Polish stocks and find supporting evidence as found for U.S markets. In a similar vein, Pyun, Lee, and Nam (2000) provide supporting evidences for the Korean stock markets as well. However, Zarraga (2003) conducts a direct test of MDH using Spanish stock returns and documents no supporting evidence for the MDH in the Spanish stock market. Similarly, Lucey (2005) documents mixed evidence which weakly supports the MDH for the Irish market. In addition, Tauchen and Pitts (1983) theoretically show that MDH hypothesis can explain why volatility is positively correlated with trading volume.

On the other hand, Smirlock and Starks (1988) examine the lead-lag relation between volatility and volume for a sample of 300 firms in New York Stock Exchange and document a significant lagged relationship between volatility and volume supporting sequential information arrival rather than simultaneous process. In a similar vein, Darrat, Rahman and Zhong (2003) examine both the contemporaneous and the lead-lag relation between volatility and volume for 30 stocks making up the Dow Jones Industrial Average (DJIA). They find no evidence for contemporaneous relation between volatility and volume for the majority of the DJIA stocks (27 out of 30). Instead, the study documents significant lead-lag (causal) relations between these two variables which supports the SIAH instead of MDH. In a similar vein, Gwilym, McMillan and Speight (2010) investigate the intraday behavior of five-minute FTSE-100, Short Sterling and Long Gilt LIFFE futures returns. The study finds strong evidence of bi-directional causality between volume and volatility on the basis of Granger causality tests which supports the SIAH. Mougoué and Aggarwal (2011) test the relationship between volatility and trading volume using data for three major currency futures contracts denominated in US dollars, namely the British pound, the Canadian dollar and the Japanese yen. The study documents significant lead-lag relations between trading volumes and return volatility which is in line with the SIAH. Similarly, Shen, Li and Zhang (2018) investigate the relationship between volume and volatility for the Chinese stock market and find no evidence that volatility and volume have a contemporaneous correlation. Instead, they document a significant lead-lag relation between volatility and volume supporting the SIAH.

The relation between volatility and volume is also analyzed using the Turkish data set as well. For example, Okan, Olgun and Takmaz (2009) investigate the relationship for the BIST-30 index futures and document consistent evidence with SIAH, but the study rejects the MDH for the BIST-30 index futures. Similarly, Kıran (2010) examines the volume-volatility relation for the BIST-100 index, but document no supporting evidence for both the MDH and SIAH in the Turkish stock market. In a similar vein, Boyacıoğlu, Güvenek and Alptekin (2010) analyze the relationship between volatility and volume for the BIST-100 index and find no evidence that supports the MDH or SIAH in Borsa Istanbul. Furthermore, Çelik (2013) investigates the volatility-volume relation for the BIST-30 index data in terms of the MDH and SIAH hypothesis and finds that while she documents supporting evidence for the MDH in pre-crisis period she finds no evidence for the MDH in crisis period. The study also fails to document strong evidence against the SIAH in crisis period.

It is clear from the above literature that no study has investigated the relation between trading volume and volatility with respect to company size. On this basis, our study is the first that investigates whether volume-volatility relationship differs between small-cap and large-cap stocks. In addition, we also analyze the relation between volume and volatility at different states of the economy which has not been analyzed before.

## 3. Data

Borsa Istanbul calculates various indices, namely BIST indices, to follow the movements in the markets. We choose three sub-indexes to represent large-cap, mid-cap small-cap stocks. Large-cap stocks represent BIST 30 index that includes largest top 30 stocks in Borsa Istanbul. Mid-cap stocks represent the BIST 100-30 index, which includes the first 100 companies, excluding the first 30 stocks, in Borsa Istanbul. Small-cap stocks represent BIST all share – 100 index which includes all listed companies in Borsa Istanbul, excluding the top 100 stocks.[2] Daily data of closing prices and trading volumes of these three sub-indices are obtained from Borsa Istanbul. Our observation period starts on January 2, 2009 and ends on October 31, 2020 and consists of 2932 daily observations. We also exclude public holidays from our data set. The beginning of our sample is purely driven by the availability of the BIST 100-30 and BIST all share - 100 indices. To compute the daily index returns we take first difference of the logarithmic form of price indices. Similar with the earlier studies (e.g., Lamoureux & Lastrapes, 1990; Gallo & Pacini, 2000; Darrat, Rahman & Zhong, 2003) we use the number of shares traded as trading volumes. Daily trading volume in logarithmic form is used as trading volume. Previous studies document evidence for both linear and nonlinear trends in the trading volume series (see, e.g., Gallant, Rossi & Tauchen, 1992; Chen, Firth & Rui, 2001). Thus, we run the below specification to test the linear and nonlinear time trends:

$$Volume_t = a_0 + a_1 t + a_2 t^2 + \varepsilon_t \tag{1}$$

$Volume_t$ is the natural log of the trading volume series. Linear and quadratic time trends are shown with $t$ and $t^2$ respectively. $a_0$, $a_1$ and $a_2$ are the coefficients. The coefficients, a1 and a2, are found to be statistically significant in our regressions for all three trading volume series. Further, using Augmented Dickey-Fuller procedures, we conduct unit root test for the residuals of these regressions, which is called the detrended volume series, and find that all the residuals are stationary.[3] This detrended volume series will be used as trading volume in the subsequent analyses.

## 4. Methodology

As is well known, financial time series present various stylized facts (such as fat tails, asymmetry and volatility clustering). Thus, several conditional volatility models have been proposed to capture those stylized facts. The GARCH (Generalized Autoregressive Conditional Heteroscedasticity) model has become very popular among these models due to being capable of explaining many of those stylized facts. Thus, previous studies mostly employed GARCH models to analyze the volume-volatility relation.

In general, the conditional variance of returns is modeled as a GARCH (1,1) process which is usually adequate to obtain a good model fit. However, if the chosen model is not correctly specified it leads to invalid inferences. The distribution of equity returns is well-known to have fatter tails. In order to capture the fat tails this study employs the Skewed Generalized Error Distribution (SGED) of Theodossiou (2000), which allows returns innovation to follow a flexible treatment of both skewness and leptokurtosis in the conditional distribution of returns.

As a second step, we search for several specifications for the conditional variance equation which best fits the data. We experiment various models such as GARCH, TARCH, IGARCH, FIGARCH and EGARCH models to find the best-fitting model according to information criteria, log likelihood values and diagnostic checks. For example, If the model is adequate, the standardized residuals should be serially uncorrelated (if the mean model is chosen correctly), and their squares should be as well (if the variance model is chosen correctly).[4] Taking into account all the criteria, we find the EGARCH (exponential generalized autoregressive conditional heteroscedasticity) model as the best-fitting model. Indeed, we find that the fit of our model is improved when the SGED is used. We obtain noticeably higher log likelihood value with the skewed generalized error distribution compared to gaussian error or the student's t distribution. This model also has some advantages when compared to pure GARCH models. As mentioned above, one of the well-known facts about volatility is the negative correlation with its lagged returns. However, pure GARCH models enforce a symmetric volatility response to negative and positive shocks. The sign is lost due to squared lagged error in the variance equation. On this basis, the EGARCH model accounts for asymmetry in volatility of return by imposing no positive constraints on estimated parameters. Our preliminary analysis also shows asymmetry in volatility persistence implying that the effect of bad news and good news on volatility persistence is not symmetric. Thus, to estimate conditional volatility, we employ EGARCH model which is specified as follows:

$$r_{j,t} = c_j + \varepsilon_{j,t},$$

$$\ln\left(h_{j,t}^2\right) = \omega_j + \beta_j \ln\left(h_{j,t-1}^2\right) + \lambda \frac{\varepsilon_{j,t-1}}{\sqrt{h_{j,t-1}^2}} + \alpha \left[\frac{|\varepsilon_{j,t-1}|}{\sqrt{h_{j,t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \tag{2}$$

Where $r_{j,t}$ represents the market index return at time t, where subscript j ∈ {BIST 30 index, BIST 100-30 index, BIST ALL SHARE –100 Index). $h_{j,t}^2$ is the conditional variance of the unexpected return series. $\omega_j$ is the long-term average volatility of related series. $\omega, \beta, \gamma\ and\ \alpha$ are parameters to be estimated.

In Equation 2, the shocks are allowed to have an asymmetric effect on the conditional variance with the coefficient λ. If λ ≠ 0, the effect is asymmetric and the leverage effect exists if λ <0. The coefficient β measures the persistence of conditional variance.

EGARCH-SGED model:

Following Theodossiou (2000), the probability density function for the Skewed Generalized Error Distribution (SGED) can be represented as follows:

$$f(\varepsilon_t) = Cexp\left(-\frac{|\varepsilon_t + \delta|^{\kappa}}{[1 + sign(\varepsilon_t + \delta)\lambda]^{\kappa}\theta^{\kappa}}\right)$$

Where

$$C = \frac{\kappa}{2\theta}\Gamma\left(\frac{1}{\kappa}\right)^{-1} ,\theta = \Gamma\left(\frac{1}{\kappa}\right)^{0.5}\Gamma\left(\frac{3}{\kappa}\right)^{-0.5} S(\lambda)^{-1} ,$$

$$S(\lambda) = \sqrt{1 + 3\lambda^2 - 4A^2\lambda^2} ,\delta = \frac{2\lambda A}{S(\lambda)},$$

$$A = \Gamma\left(\frac{2}{\kappa}\right)\Gamma\left(\frac{1}{\kappa}\right)^{-0.5}\Gamma\left(\frac{3}{\kappa}\right)^{-0.5}.$$

Where the shape parameter $\kappa$ controls the height and tails of the density function with constraint $\kappa > 0$. $\lambda$ is a skewness parameter of the density function obeying the following constraint $-1 < \lambda < 1$. In the case of negative (positive) skewness, the density function is skewed to the left (right). Sign is the sign function and $\Gamma(a) = \int_0^{+\infty} z^{a-1}e^{z}dz$ is the gamma function. When $\kappa = 2$ $and$ $\lambda = 0$ the SGED distribution turns out to be the standard normal distribution. Smaller values of κ have fatter tails.

According to Efficient market hypothesis, asset prices increase or decrease only in response to new information. However, rate of information flow is not possible to be observed; thus, trading volume is suggested to be a proxy for unobservable information arrival (Lamoureux & Lastrapes, 1990; Anderson, 1996; Arago & Nieto, 2005).

In this context, most earlier studies have tested MDH hypothesis in such a way that contemporaneous trading volume is included as an exogenous explanatory variable in the conditional variance equation (typically with GARCH type models). However, as both volume and volatility are endogeneously determined it leads to an issue of simultaneity bias. To overcome the simultaneity bias, this study employs the generalized method of moments (GMM) approach, following Mougoue and Aggarwal (2011), which fixes simultaneity issue in our estimations and provides heteroskedasticity consistent standard errors. To this end, to test the MDH hypothesis, we use lagged values of endogenous variables as instruments and estimate the system of equations as follows:

$$h_{j,t}^2 = \alpha_0 + \alpha_1 Vol_{j,t} + \alpha_2 Vol_{j,t-1} + \alpha_3 h_{j,t-1}^2 + v_{h,t} \tag{3}$$

$$Vol_{j,t} = \eta_0 + \eta_1 h_{j,t}^2 + \eta_2 h_{j,t-1}^2 + \eta_3 Vol_{j,t-1} + v_{vol,t} \tag{4}$$

In equations 3 and 4, $h^2_t$ is the conditional volatility obtained through the EGARCH-SGED process.[5] $Vol_t$ is the detrended trading volume series. Our interest is focused on the sign and statistical significance of the coefficients α1 and η1. MDH suggests that trading volume is a useful proxy for unobservable information flows. If this is the case, then α1 should be found positive and significant.

Specifications 3 and 4 assume that the impact of volume on volatility is constant over different states of the economy. As mentioned before, it is possible that the relation between volume and volatility can change across low and high growth periods of the economy. To this end, our model is specified to allow the impacts of volume on volatility to vary across different stages of the business cycle. To accomplish this, dummy variables is used to capture recession and expansion periods.

$$h_t^2 = \xi_0 + \xi_1 Rec_t Vol_t + \xi_2 Ex_t Vol_t + \xi_3 Rec_{t-1} Vol_{t-1} + \xi_4 Ex_{t-1} Vol_{t-1} + \xi_5 h_{t-1}^2 + v_{h,t} \quad (5)$$

$$Rec_t Vol_t = \theta_0 + \theta_1 h_t^2 + \theta_2 h_{t-1}^2 + \theta_3 Rec_{t-1} Vol_{t-1} + v_{rv,t} \quad (6)$$

$$Ex_t Vol_t = \delta_0 + \delta_1 h_t^2 + \delta_2 h_{t-1}^2 + \delta_3 Ex_{t-1} Vol_{t-1} + v_{ev,t} \quad (7)$$

Where, Rec = 1 if the economy is in a recessionary state at time t, and zero otherwise; Exp = 1 if the economy is in an expansionary state at time t, and zero otherwise. All the other variables and coefficients are defined similarly as in specification 3 and 4.[6]

To test for the lead-lag relation between volatility and volume, we adapt a VAR model to test for granger causality. Optimal lag is selected according to Schwarz Information Criteria (SIC). Followig Chiang, Qiao and Wong (2010) a VAR(p) model is estimated as the following:[7]

$$h_t^2 = c_1 + \sum_{i=1}^{k} \Psi_i \, h_{t-i}^2 + \sum_{i=1}^{k} \Omega_i \, Vol_{t-i} + \varepsilon_{1t} \quad (8)$$

$$Vol_t = c_2 + \sum_{i=1}^{k} \Pi_i \, h_{t-i}^2 + \sum_{i=1}^{k} \vartheta_i \, Vol_{t-i} + \varepsilon_{2t} \quad (9)$$

Where, $c_1$ and $c_2$ are the intercepts, $\Psi_i, \Omega_i, \Pi_i$ and $\vartheta_i$ are the parameters to be estimated, k denotes the optimal lag lengths obtained by using SIC. All the other variables are defined similarly as in equation (4). We can test whether the trading volume granger causes return volatility by the null hypothesis that $H_0$: $\Omega_i$=0 for all i = 1, 2, …, k. If a standard F-test can reject the null hypothesis that $\Omega_i$ = 0 for all i, then trading volume granger causes volatility. In a similar vein, volatility granger causes trading volume if the null hypothesis that $H_0$: $\Pi_i$= 0 for all i can be rejected with F-test at conventional significance levels. Since the predictability of volatility is of great importance to investors our interest is focused on causal relation from volume to volatility.

When the volume-volatility relationship in the context of SIAH hypotheses is tested during recession and expansion periods, the detrended trading volume series, $Vol_t$ is replaced with the $Rec_t Vol_t$ and $Exp_t Vol_t$ series to capture recession and expansion periods as defined previously and thus equations 10 through 12 are re-estimated as below:

$$h_t^2 = c_1 + \sum_{i=1}^{k} \phi_i \, h_{t-i}^2 + \sum_{i=1}^{k} \Omega_i^{Rec} Rec_{t-i} Vol_{t-i} + \sum_{i=1}^{k} \Omega_i^{Ex} \, Ex_{t-i} Vol_{t-i} + \varepsilon_{1t} \quad (10)$$

$$Rec_t Vol_t = c_2 + \sum_{i=1}^{k} \lambda_i \, h_{t-i}^2 + \sum_{i=1}^{k} \Lambda_i^{Rec} Rec_{t-i} Vol_{t-i} + \sum_{i=1}^{k} \Lambda_i^{Ex} \, Ex_{t-i} Vol_{t-i} + \varepsilon_{1t} \quad (11)$$

$$Ex_t Vol_t = c_3 + \sum_{i=1}^{k} \varphi_i \, h_{t-i}^2 + \sum_{i=1}^{k} \kappa_i^{Rec} Rec_{t-i} Vol_{t-i} + \sum_{i=1}^{k} \kappa_i^{Ex} \, Ex_{t-i} Vol_{t-i} + \varepsilon_{1t} \quad (12)$$

Where, $c_1$, $c_2$ and $c_3$ are the intercepts, $\phi_i, \Omega_i^{Rec}, \Omega_i^{Ex}, \lambda_i, \Lambda_i^{Rec}, \Lambda_i^{Ex}, \varphi_i$, $\kappa_i^{Rec}$and $\kappa_i^{Ex}$ are the parameters to be estimated, k denotes the optimal lag lengths obtained by using SIC. All the other variables are defined similarly as in equation 8 and 9.

We can test whether volume granger causes volatility during recession periods by the null hypothesis that $H_0$: $\Omega_i^{Rec} = 0$ for all i = 1, 2, …, k. If a standard F-test can reject the null hypothesis that $\Omega_i^{Rec} = 0$ for all i, then volume granger causes volatility in recession periods. Similarly, we can test whether volume granger causes volatility during expansion periods by the null hypothesis that $H_0$: $\Omega_i^{Ex} = 0$ for all i = 1, 2, …, k. If a standard F-test can reject the null hypothesis that $\Omega_i^{Ex} = 0$ for all i, then trading volume granger causes volatility in expansion periods. In a similar vein, volatility granger causes trading volume in recession periods if the null hypothesis that $H_0$: $\lambda_i = 0$ for all i can be rejected with F-test at conventional significance levels. Similarly, we can test whether volatility granger causes trading volume in expansion periods if the null hypothesis that $H_0$: $\varphi_i = 0$ for all i can be rejected with standard F-test.

## 5. Descriptive Statistics

Before beginning the estimation of our equations, we also provide detail summary statistics for daily returns and trading volumes. When looking at the mean returns and related standard deviations for large-cap, mid-cap and small-cap stocks in Table 1 it is seen that standard deviation of large cap stock returns relative to its mean is very high compared to the standard deviation of mid cap and small cap stock returns. For example, in terms of relation between risk and return, while the standard deviation is roughly 33 times higher than its mean for large cap stocks it is 19 and 13 times higher than its mean for the mid-cap and small-cap stocks respectively. According to the risk-return trade-off, expected return of large cap stocks is not high enough to justify its risk compared to the expected return of mid cap and small cap stocks.

**Table 1**. Descriptive Statistics for Daily Returns and Raw Trading Volumes

| | ***Large cap*** | | ***Mid cap*** | | ***Small cap*** | |
|---|---|---|---|---|---|---|
| | $R_t$ | $V_t$ | $R_t$ | $V_t$ | $R_t$ | $V_t$ |
| Mean | 0.00046 | 20.0023 | 0.00069 | 19.3070 | 0.00092 | 19.159 |
| Median | 0.00081 | 19.9293 | 0.00160 | 19.2407 | 0.00175 | 18.913 |
| Maximum | 0.06965 | 22.3127 | 0.08802 | 21.8094 | 0.08821 | 22.045 |
| Minimum | -0.1090 | 17.8182 | -0.1179 | 16.8178 | -0.1324 | 17.097 |
| SD | 0.01555 | 0.56869 | 0.01319 | 0.74101 | 0.01226 | 0.8432 |
| Skewness | -0.0448 | 0.61169 | -1.3616 | 0.59562 | -1.6879 | 1.2883 |
| Kurtosis | 6.2776 | 4.6423 | 12.978 | 3.2006 | 1.8914 | 4.683 |
| Observations | 2931 | 2932 | 2931 | 2932 | 2931 | 2932 |

**Note:** this Table shows descriptive statistics for the large-cap, mid-cap and small-cap stocks. $R_t$ and $V_t$ denote daily returns and trading volumes respectively. The sample covers 2 January 2009 to 31 October 2020.

When we look at the mean volumes and their related standard deviations for equities of large-cap, mid-cap and small-cap firms we come across with quite a different picture. While the standard deviation of volume is roughly 35 times lower than its mean volume for large-cap stocks it is 26 and 22 times lower than its mean volume for the mid-cap and small-cap stocks respectively. In other words, the volatility of volume for small cap stocks is higher than the volatility of volume for large cap stocks which is just as expected. When looking at skewness values for daily returns it is seen that the distributions of daily returns for large cap, mid cap and small cap stocks are negatively skewed, while the distributions of volume series are positively skewed which indicate that they are non-symmetric, but since the value of -0.04 for the return of large cap stocks is quite close to zero the return distribution large cap stocks can be accepted as symmetrical. When we look at the kurtosis values for daily returns, we observe high values such as (6.27 and 12.97) for large cap and midcap stocks respectively. Since the kurtosis values exceed 3, both distributions are peaked (leptokurtic) relative to the normal distribution. However, small cap stock returns exhibit low level of kurtosis with a value of 1.89 which indicates a flat (platykurtic) distribution relative to the normal since the value is less than 3. When looking at kurtosis values for daily volume for the stocks of large-cap, mid-cap and small-cap firms it is seen that they are moderately leptokurtic relative to normal distribution.

## 6. Results

Table 2 shows estimation results of equations 3 and 4 in which we test the MDH hypothesis. We are mainly interested in the sign and statistical significance of the coefficients α1 and η1. More specifically, if trading volume is a useful proxy for unobservable information flows, as suggested by MDH, then α1 should be found positive and significant.

**Table 2.** Contemporaneous Relation Between Volume and Volatility

| | Large-Cap | Mid-Cap | Small-Cap |
|---|---|---|---|
| $\alpha_i$ | 0.3393 | 0.0517 | -0.137 |
| | (6.56)*** | (0.49) | (-1.18) |
| $\eta_i$ | 0.1002 | 0.0026 | -0.0059 |
| | (3.89)*** | (0.59) | (0.09) |

This Table shows the GMM estimates of contemporaneous terms for specification (3) and (4). The exogenous terms are excluded. The sample period starts on January 2 2009 and ends on October 31 2020. The t-statistical values are shown in parentheses. ** and *** show significance at 5 and 1 percent levels, respectively.

When looking at the Table 2 it is seen that the coefficients α1 and η1 are only found to be significant for large-Cap stocks. In other words, while we document a significant contemporaneous relation between volatility and volume for large cap stocks, we find no evidence of contemporaneous relation between volatility of return and trading volume for the mid-cap and small-cap stocks. This finding is somehow consistent with the asymmetric trading response hypothesis of McQueen et al. (1996). They propose that investors sell all types of firms in the event of bad news but buy only the large firms during good news. Nofsinger (2001) documents supporting evidence and finds that investors react quickly to good news by buying large firms but not small firms. On this basis, our findings suggest that the dissemination of information in the stock market appears to be primarily through large firms. In other words, information is first incorporated into the prices of large companies, most probably thanks to foreign and local institutional investors. Furthermore, our finding is not consistent with the findings of previous studies that analyze Turkish stock market. For example, Okan et al. (2009) reject the MDH hypothesis for the BIST-30 index futures while we find supporting evidence. The difference can be attributed to the either studied time period or applied methodology. Regarding the time period, Okan et al. (2009) analyze the BIST-30 index futures for the period between 2006 and 2008, whereas we examine the BIST 30 index for the period between 2009 and 2020. Regarding the latter, Okan et al. (2009) test the MDH hypothesis by including contemporaneous trading volume as an exogenous explanatory variable in the conditional variance equation of GARCH-type models. However, since both volume and volatility can be endogeneously determined it might lead to an issue of simultaneity bias. Unlike their study, in order to obtain unbiased and consistent estimates, our study employs the generalized method of moments (GMM) approach.

In a similar vein, unlike our findings, studies such as Kıran (2010) and Boyacıoğlu et al. (2010) test the relationship between trading volume and volatility for the BIST 100 index and document no supporting evidence for the MDH in Borsa Istanbul.

Besides the two reasons stated above, we can also attribute the difference of our results with the results of these two studies to the analyzed indices. While BIST 100 index includes the top 100 companies in Borsa Istanbul, BIST 30 index includes the top 30 companies in Borsa Istanbul. On this basis, comparison of these results is most likely to be inaccurate.

In addition, as mentioned previously, the state of the economy can also have different impacts on the relation between return volatility and trading volume. The contemporaneous interaction between volume and volatility is assumed to be constant in specification 3 and 4. Specification 5 allows the effects of volume on volatility to vary over different states of the economy. In a similar vein, in Specification 6 and 7, the impact of volatility on volume is allowed to vary over different stages of the business cycle. Table 3 presents the results of estimations in equations 5 through 7.

**Table 3.** Contemporaneous Relation Between Volume and Volatility Conditional on State of the Economy

| | Large-Cap | Mid-Cap | Small-Cap |
|---|---|---|---|
| $\xi_1$ | 0.282 | 0.033 | -0.231 |
| | (2.92)*** | (0.16) | (-1.03) |
| $\xi_2$ | 0.3684 | 0.0635 | -0.089 |
| | (6.36)*** | (0.52) | (-0.84) |
| $\theta_1$ | 0.0288 | 0.0005 | -0.0035 |
| | (2.14)** | (0.16) | (-1.34) |
| $\delta_1$ | 0.0712 | 0.0002 | -0.0025 |
| | (4.44)*** | (0.54) | (-0.95) |

This Table shows the GMM estimates of contemporaneous terms for the specifications (5), (6) and (7). The exogenous terms are excluded. The sample period starts on January 2 2009 and ends on October 31 2020. The t-statistical values are shown in parentheses. **and*** show significance at 5 and 1 percent levels, respectively.

When looking at the sign and statistical significance of the coefficients $\xi_1, \xi_2, \theta_1\ and\ \delta_1$ in Table 3 it is seen that we find no statistical evidence of contemporaneous relation between volume and volatility for mid-cap and small-cap stocks. For the stocks of large-cap firms, when we divide the observation period into sub-periods, we find no evidence, that the impact of volume on contemporaneous volatility or the impact of volatility on contemporaneous volume varies across states of the market. All the coefficients are significant in both recession and expansion periods.

Having tested the relation between volume and volatility in the context of MDH hypothesis we now move to test this relation with respect to SIAH hypothesis. As mentioned previously SIAH states that volume and volatility possess a lead–lag relation but not a contemporaneous correlation. The model predicts a causal relation between volume and volatility. In order to test this lead-lag relation we employ a VAR model and test for granger causality with the equations 8 and 9. Table 4 shows Linear Granger causality tests between volume and volatility. We see that the first null hypothesis of $H_0 = \Omega_1 = \Omega_2 = \cdots \Omega_k = 0$ is not rejected at significant levels for the stocks of any sized stock indices. This implies that past information on trading volume cannot be used to forecast return volatility for the large-cap, mid-cap and small-cap stocks.

When looking at the second hypothesis of $H_0 = \Pi_1 = \Pi_2 = \cdots \Pi_k = 0$ it is seen that it is rejected for the stocks of large-cap, mid-cap and small-cap companies which implies that past information on return volatility can be used to forecast trading volume for the large-cap, mid-cap and small-cap stocks. Thus, we find no evidence of bi-directional causality between volatility and volume for Turkish stocks. On the contrary, we document unidirectional causality running from trading volume to return volatility but not the other way around which invalidates SIAH.

Overall, for all Turkish stocks, our findings contradict the prediction of SIAH.

**Table 4.** Linear Granger Causality Tests Between Trading Volume (Vol) and Return Volatility ($h^2$)

| Hypothesis | Large-Cap | Mid-Cap | Small-Cap |
|---|---|---|---|
| $H_0 : Vol \rightarrow h^2$ | 12.064 | 11.674 | 7.8251 |
| | (0.28) | (0.38) | (0.73) |
| $H_0 : h^2 \rightarrow Vol$ | 30.679 | 31.969 | 71.968 |
| | (0.00)*** | (0.00)*** | (0.000)*** |

Note: this Table reports $\chi^2$ statistics with their corresponding significance levels in parentheses. " → "means does not cause. Each variable has equal number of lags selected by the Schwarz Information Criterion. ** and *** denote statistical significance at 5% and 1% respectively.

Finally, we move to test the relationship between volume and volatility in the context of SIAH hypotheses during recession and expansion periods with the equations 10 through 12. Table 5 shows Linear Granger causality tests between volume and volatility. We see that the null hypotheses of $H_0 = \Omega_1^{Rec} = \Omega_2^{Rec} = \Omega_k^{Rec} = 0$ is not rejected for the stocks of companies of any size which indicates that trading volume does not Granger cause return volatility in recession periods. However, the null hypothesis of $H_0 = \Omega_1^{Exp} = \Omega_2^{Exp} = \Omega_k^{Exp} = 0$ is rejected at 1% significant level for large cap stocks which shows that past information on trading volume can be used to forecast return volatility for the stocks of large-cap companies.

When looking at the second hypothesis of $H_0 = \lambda_1 = \lambda_2 = \lambda_k = 0$ and $H_0 = \varphi_1 = \varphi_2 = \varphi_k = 0$ it is seen that it is rejected at 1% significant level for the stocks of companies of any size which indicates that return volatility Granger causes trading volume in both recession and expansion periods.

**Table 5.** Linear Granger Causality Tests Between Return Volatility ($h^2$) and Trading Volume (Vol) Conditional on State of the Economy

| Hypothesis | Large-Cap | Mid-Cap | Small-Cap |
|---|---|---|---|
| ***Expansion*** | | | |
| $H_0 : Vol \nrightarrow h^2$ | 31.69 | 8.70 | 13.97 |
| | (0.046)** | (0.65) | (0.87) |
| $H_0 : h^2 \nrightarrow Vol$ | 25.165 | 23.14 | 84.62 |
| | (0.005)*** | (0.016)** | (0.000)*** |
| ***Recession*** | | | |
| $H_0 : Vol \nrightarrow h^2$ | 17.81 | 15.44 | 16.099 |
| | (0.60) | (0.16) | (0.76) |
| $H_0 : h^2 \nrightarrow Vol$ | 22.847 | 21.95 | 31.52 |
| | (0.003)*** | (0.025)** | (0.065) |

This Table reports $\chi^2$ statistics with their corresponding significance levels in parentheses. " $\nrightarrow$ "means does not cause. Each variable has equal number of lags selected by the Schwarz Information Criterion. ** and *** denote statistical significance at 5% and 1% respectively.

To sum up, for the stocks of large-cap companies, we document strong evidence of bi-directional causality between volume and volatility in expansion periods which supports the SIAH, whereas we document uni-directional causality, runs from volatility to volume but not the other way around, in recession periods which is contrary to the SIAH. For mid-cap stocks, we document uni-directional causality running from volatility to volume but not the other way around in both recession and expansion periods which is not in line with the prediction of SIAH. For small-cap stocks, we document uni-directional causality, runs from volatility to volume but not the other way around only in expansion periods which is also contrary to the SIAH.

## 7. Conclusion

The correlation between volatility of stock return and trading volume has long been a subject of research in finance. Various theoretical models have attempted to address this relationship and two basic approaches receive widespread attention. The first one is the MDH which states that the trading volume and volatility contemporaneously change in response to new information, since information dissemination is contemporaneous. Hence, volume and volatility does not possess a lead–lag relation but a contemporaneous correlation. The second one is the SIAH which predicts a causal relation between return volatility and volume. It states that the formation of equilibrium is not instantaneous and requires some time which produces a lead-lag relation between trading volume and volatility.

Our study tests the relationship between volume and volatility in the context of the MDH and SIAH hypotheses for large-cap, mid-cap and small-cap stocks in Borsa Istanbul. We find a significant contemporaneous relation between volume and volatility for large cap stocks, whereas we document no evidence of contemporaneous relation between volume and volatility for mid-Cap and small-cap stocks. In this sense, our findings for large-cap stocks are consistent with the prediction of MDH which suggests that information appears to be primarily incorporated into the prices of large companies. When testing the MDH across different economic states it is seen that our findings have not changed.

In addition, our findings for the stocks of companies of any size show strong evidence of uni-directional causality running from volatility to volume but not the other way around which is not consistent with the SIAH.

When we divide the observation period into sub-periods, for large-cap stocks, we document bi-directional causality between volume and volatility in expansion periods which is in line with the SIAH, whereas we document an evidence of only uni-directional causality running from volatility to volume but not the other way around in recession periods which invalidates SIAH. For mid-cap stocks, we find unidirectional causality running from volatility to volume but not the other way around in both recession and expansion periods. For small-cap stocks, we find unidirectional causality running from volatility to volume but not the other way around only in expansion periods.

As a result, during our observation period, when the company size is taken into consideration, the relation between volume and volatility is found to be slightly different than previously recognized.

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## End Notes

1. *There is a vast literature on investor trading behavior which finds that foreign investors and local institutional investors prefer large firms compared to small firms due to their liquidity concerns (See, e.g., Dahlquist & Robertsson, 2001; Covrig et al., 2006).*
2. *Companies are categorized based on their size — large-cap, midcap, and small-cap. Cap is short for market capitalization, which is calculated by multiplying the stock's current price by the total number of outstanding shares. Stocks are assigned to one of three size (market-capitalization) portfolios every three months based on the prior quarter's ending prices and shares outstanding.*
3. *The results are not presented here for the sake of brevity, but they are available upon request.*
4. *To produce serially uncorrelated residuals, which shows that the mean model is specified correctly, we include 3 lags and 5 lags of the dependent variable for the series of mid-cap stocks and small-cap stocks respectively.*
5. *Estimations of the EGARCH-SGED models and diagnostic test results related to the assumptions of the models can be found in Appendix.*
6. *The recession and expansion periods of the Turkish economy are obtained from the website of an international organization called OECD (The Organization for Economic Co-operation and Development) which provides turning points in the growth cycle for OECD countries. http://www.oecd.org/std/leadingindicators/oecdcompositeleadingindicatorsreferenceturningpointsandcomponentseries.htm*

7. *However, when the errors of the VAR(p) model is tested for autocorrelation, we find that the proposed criteria for lag selection do not guarantee uncorrelated errors. Therefore, we add extra lags to solve the autocorrelation problem. However, the granger causality test results are qualitatively same with the extra lags compared to the results of lags proposed by the information criteria. The number of lags needed to solve the autocorrelation problem and the diagnostic test results related to the assumptions of the model is reported in the Appendix.*

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## Appendix

**Table 6.** EGARCH-SGED Model Estimation of BIST 30 Index (Large-cap return)

| | | |
|---|---|---|
| **Panel A:** | | |
| **Variable** | **Coeff** | **T-Stat** |
| Mean (Return) | 0.1868 | 4.47 |
| C | 0.1021 | 3.67 |
| A | 0.0302 | 2.55 |
| B | 0.8824 | 41.26 |
| D | 0.0742 | 3.68 |
| %SKEWGED_K | 1.4019 | 28.69 |
| %SKEWGED_LAMBDA | -0.0631 | -3.019 |
| **Panel B:** | | |
| *Ljung-Box Q-Statistics (Std. Res)* | | |
| Lags | Statistic | Significance level |
| 2 | 2.486 | 0.28 |
| 4 | 3.540 | 0.47 |
| 6 | 5.009 | 0.54 |
| 8 | 5.139 | 0.74 |
| 10 | 7.894 | 0.64 |
| 12 | 15.997 | 0.19 |
| **Panel C:** | | |
| *Ljung-Box Q-Statistics (Std.Res$^2$)* | | |
| Lags | Statistic | Significance level |
| 2 | 2.335 | 0.31 |
| 4 | 7.248 | 0.123 |
| 6 | 10.006 | 0.124 |
| 8 | 10.143 | 0.255 |
| 10 | 11.040 | 0.354 |
| 12 | 11.744 | 0.466 |
| Log Likelihood -5235 | | |

Note: Table 6 shows the estimations of EGARCH-SGED Model for the BIST 30 Index (Large-cap return). In Panel A, the first column reports variables. Coefficients and related *t* statistics for corresponding variables are also shown in the second and third columns respectively. The constant coefficient in the mean equation is labelled as Mean. Constant in the variance equation is labeled as C. “arch” (lagged squared residuals) parameter is labeled as A. “garch” (lagged variance) parameter is labeled as B. Asymmetry coefficient is labeled as D. %SKEWGED_K is the shape parameter that controls the height and tails of the density function with a constraint κ>0. %SKEWGED_LAMBDA is the skewness parameter of the density function with a constraint -1<λ<1. Panel B reports Ljung-Box Q-Statistics test up to 12 lags for the standardized residuals. Statistics of the test and related significance levels are reported in the second and third columns respectively. Panel C reports Ljung-Box Q-Statistics test up to 12 lags for the Squared Residuals. Statistics of the test and related significance levels are reported in the second and third columns respectively. Log likelihood value is also reported at the bottom of the table. The sample period starts on January 2 2009 and ends on October 31 2020.

**Table 7.** EGARCH-SGED Model Estimation of BIST 100-30 Index (mid-cap return)

| Variable | Coeff | T-Stat |
|---|---|---|
| Constant | 0.3205 | 13.85 |
| Midreturn{1} | 0.0453 | 2.319 |
| Midreturn{2} | 0.0240 | 1.56 |
| Midreturn{3} | -0.0313 | -2.084 |
| C | 0.1891 | 6.058 |
| A | 0.0643 | 2.323 |
| B | 0.7188 | 20.97 |
| D | 0.1343 | 3.454 |
| %SKEWGED_K | 1.1608 | 33.69 |
| %SKEWGED_LAMBDA | -0.1586 | -9.97 |
| *Ljung-Box Q-Statistics (Std. Res)* | | |
| Lags | Statistic | Significance level |
| 2 | 3.706 | 0.156 |
| 4 | 6.417 | 0.17 |
| 6 | 6.87 | 0.33 |
| 8 | 8.93 | 0.35 |
| 10 | 11.67 | 0.31 |
| 12 | 18.66 | 0.10 |
| *Ljung-Box Q-Statistics (Std.Res$^2$)* | | |
| Lags | Statistic | Significance level |
| 2 | 0.515 | 0.77 |
| 4 | 1.06 | 0.90 |
| 6 | 1.37 | 0.967 |
| 8 | 1.786 | 0.986 |
| 10 | 2.195 | 0.994 |
| 12 | 2.616 | 0.997 |
| Log Likelihood: - 4446 | | |

Note: Table 7 shows the estimations of EGARCH-SGED Model for the BIST 100-30 Index (Mid-cap return). In Panel A, the first column reports variables. Coefficients and related t statistics for corresponding variables are also shown in the second and third columns respectively. The constant coefficient in the mean equation is labelled as Mean. Midreturn{1}, Midreturn{2}, and Midreturn{3} represents the first three lags of the dependent variable in the mean equation. Constant in the variance equation is labeled as C. “arch” (lagged squared residuals) parameter is labeled as A. “garch” (lagged variance) parameter is labeled as B. Asymmetry coefficient is labeled as D. %SKEWGED_K is the shape parameter that controls the height and tails of the density function with a constraint $\kappa>0$. %SKEWGED_LAMBDA is the skewness parameter of the density function with a constraint $-1<\lambda<1$. Panel B resports Ljung-Box Q-Statistics test up to 12 lags for the standardized residuals. Statistics of the test and related significance levels are reported in the second and third columns respectively. Panel C reports Ljung-Box Q-Statistics test up to 12 lags for the Squared Residuals. Statistics of the test and related significance levels are reported in the second and third columns respectively. Log likelihood value is also reported at the bottom of the table. The sample period starts on January 2 2009 and ends on October 31 2020.

**Table 8.** EGARCH-SGED Model Estimation of BIST ALL- 100 Index (Small Cap return)

| | | |
|---|---|---|
| **Panel A:** | | |
| **Variable** | **Coeff** | **T-Stat** |
| Constant | 0.2491 | 33.46 |
| Smallreturn{1} | 0.0754 | 4.57 |
| Smallreturn{2} | 0.0346 | 3.48 |
| Smallreturn{3} | 0.0367 | 2.69 |
| Smallreturn{4} | 0.0559 | 3.88 |
| Smallreturn{5} | 0.0228 | 2.29 |
| C | 0.0699 | 4.81 |
| A | 0.147 | 4.82 |
| B | 0.7496 | 25.52 |
| D | 0.0805 | 2.43 |
| %SKEWGED_K | 1.1327 | 42.90 |
| %SKEWGED_LAMBDA | -0.1545 | -11.67 |
| **Panel B:** | | |
| *Ljung-Box Q-Statistics (Std. Res)* | | |
| Lags | Statistic | Significance level |
| 2 | 1.740 | 0.418 |
| 4 | 2.698 | 0.61 |
| 6 | 3.424 | 0.75 |
| 8 | 4.718 | 0.79 |
| 10 | 15.89 | 0.196 |
| 12 | | |
| **Panel C:** | | |
| *Ljung-Box Q-Statistics (Std.Res$^2$)* | | |
| Lags | Statistic | Significance level |
| 2 | 0.120 | 0.94 |
| 4 | 1.386 | 0.846 |
| 6 | 2.365 | 0.883 |
| 8 | 3.528 | 0.896 |
| 10 | 3.721 | 0.959 |
| 12 | 3.899 | 0.985 |
| Log Likelihood -3877 | | |

Note: Table 8 shows the estimations of EGARCH-SGED Model for the BIST ALL - 100 Index (Small-cap return). In Panel A, the first column reports variables. Coefficients and related t statistics for corresponding variables are also shown in the second and third columns respectively. The constant coefficient in the mean equation is labelled as Mean. Smallreturn{1}, Smallreturn{2}, and Smallreturn{3}, Smallreturn{4}, and Smallreturn{5} represents the first five lags of the dependent variable in the mean equation. Constant in the variance equation is labeled as C. “arch” (lagged squared residuals) parameter is labeled as A. “garch” (lagged variance) parameter is labeled as B. Asymmetry coefficient is labeled as D. %SKEWGED_K is the shape parameter that controls the height and tails of the density function with a constraint κ>0. %SKEWGED_LAMBDA is the skewness parameter of the density function with a constraint -1<λ<1. Panel B resports Ljung-Box Q-Statistics test up to 12 lags for the standardized residuals. Statistics of the test and related significance levels are reported in the second and third columns respectively. Panel C reports Ljung-Box Q-Statistics test up to 12 lags for the Squared Residuals. Statistics of the test and related significance levels are reported in the second and third columns respectively. Log likelihood value is also reported at the bottom of the table. The sample period starts on January 2 2009 and ends on October 31 2020.

**Table 9.** VAR Residual Portmanteau Tests for Autocorrelations

| | | | |
|---|---|---|---|
| $w_1 = [h_L^2, Vol_L]$ | Lags | Adj Q-Stat | Prob.* |
| | 11 | 7.8136 | 0.1 |
| $w_2 = [h_M^2, Vol_M]$ | Lags | Adj Q-Stat | Prob.* |
| | 12 | 6.5739 | 0.1602 |
| $w_3 = [h_S^2, Vol_S]$ | Lags | Adj Q-Stat | Prob.* |
| | 12 | 6.4746 | 0.166 |
| $w_4 = [h_L^2, RecVol_L, Exvol_L]$ | Lags | Adj Q-Stat | Prob.* |
| | 21 | 13.1259 | 0.157 |
| $w_5 = [h_M^2, RecVol_M, Exvol_M]$ | Lags | Adj Q-Stat | Prob.* |
| | 12 | 11.746 | 0.228 |
| $w_6 = [h_S^2, RecVol_S, Exvol_S]$ | Lags | Adj Q-Stat | Prob.* |
| | 22 | 11.847 | 0.22 |

Note: Table 9 shows VAR Residual Portmanteau Tests for Autocorrelations. The Null Hypothesis: No residual autocorrelations up to lag h. Test is valid only for lags larger than the VAR lag order. Fist column shows VAR systems. For example, first three rows in the first column show bivariate VAR system that includes volatility and detrended volume series for three indices. The last three rows show Trivariate VAR system that includes volatility and detrended volume series for recession and expansion periods. $Vol_t$ is replaced with the $Rec_tVol_t$ and $Exp_tVol_t$ series as defined previously. Second column shows the number of lags. The number of lags needed to solve the autocorrelation problem in the VAR system is 1 less than the number of lags reported in the test. Third and fourth columns report Adjusted Q-Statistic values and corresponding probabilities.The sample period starts on January 2 2009 and ends on October 31 2020.

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