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Education, Local Amenity and Spatial Agglomeration in a Small-Open Multi-Regional Economic Growth Model: Extending the Uzawa-Lucas Model to an Interregional Economy

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Abstract: *This paper proposes a multi-regional growth model with endogenous human and physical capital in a perfectly competitive economy. Structurally it is an extension of the Uzawa-Lucas model to an interregional economy, even though this paper models behavior of households in an alternative way and introduce Arrow's learning by doing and Zhang's creative leisure (or learning through consuming) in modeling human capital accumulation. The small-open economy consists of multiple regions and each region has production, one education and one housing sector. The production side is the same as in the neoclassical growth theory. Households move freely among regions, making decisions on levels of choosing housing, education, good, consumption and saving. A region's amenity which affects households' decisions is endogenous, depending on the region's output and population. The model describes a dynamic interdependence among wealth accumulation, human capital accumulation, time distribution (among leisure, work and education) and division of labor with endogenous wealth and income distribution among regions under perfect competition. This study also simulates the motion of the national economy and examines effects of changes in some parameters.*

Keywords: Education, human and physical capital, multiple regions, amenity, agglomeration

JEL Classification: R11

1. Introduction

Economic geography has increasingly caused attention in economic geography.[1] There are many theoretical and empirical studies for finding causal relationships among accumulated human and physical capital, natural endowments, and economic geography. Empirical studies show that economic activity and population distribution are not spatially random. As pointed out by Baldwin and Martin (2004: 2675-6), "Many of the most popular economic geography models focus on labor. … These are unsuited to the study of growth." Capital accumulation is seldom modeled with land use pattern and land markets in the literature of geographical economics. Fujita and Thisse (2002: 389) state the current situations of spatial economic growth as follows: "Clearly, space and time are intrinsically mixed in the process of economic development. However, the study of their interaction is a formidable task. … Not surprisingly, therefore, the field is still in its infancy, and relevant contributions have been few." Indeed, most of recent papers and books in the field of new economic geography are concerned with imperfect competition but neglect capital accumulation and the role of education.[2] The purpose of this study is to develop a general

[1] *For surveys on the literature, it is referred to Henderson and Thisse (2004), Capello and Nijkamp (2004), and Wu (2010).*

[2] *Obviously, capital accumulation and capital mobility are important variables for explaining spatial dynamics. Clustering of people into a single metropolitan area like Tokyo or Shanghai can hardly be explained without taking capital as endogenous variables.*

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equilibrium framework with multiple regions and human and physical capital accumulation under perfect competition. It is shown that regional differences in productivity, amenity, and resources (land) lead to differences in economic activities and population distribution.

As pointed out by Storper and Scott (2009: 147), "One of the most complex enigmas of contemporary social science concerns the causes of urban growth and associated spatial patterns of population movement." In the literature of spatial economics, to model migration over time and space in the context of regional growth and development has been a challenging question for long time. The purpose of this study is to develop a multi-regional economic growth model of free inter-regional migration with endogenous human and physical capital accumulation. The model in this study is based on some well-established economic theories. As far as mechanisms of economic growth are concerned, our model is based on the neoclassical growth theory and the Uzawa-Lucas two-sector growth model of human capital accumulation with education. It is well-known that most of the models in the neoclassical growth theory are extensions and generalizations of the pioneering works of Solow (1956).[3] The model has played an important role in the development of economic growth theory by using the neoclassical production function and neoclassical production theory. This study applies the neoclassical growth theory to a multi-regional economy. Although some attempts have been to apply neoclassical growth theory to address spatial growth issues, these models do not take account of land and regional differences in amenity.[4]

In modern economies, human capital is a key determinant of economic growth.[5] The role of human capital accumulation and utilization in economic growth is currently a main topic in economic theory and empirical research. It has become evident that it is not enough to be only concerned with capital accumulation in the neoclassical growth theory in order to explain why economies grow differently over space. According to Easterlin (1981), there were few people outside North-Western Europe and North America who had any formal education in 1850. The wide spread of formal education in school seems to have preceded the modern economic growth. Many studies have been conducted to examine dynamic interdependence between education and economic growth.

Estimating the impact of education on earnings has been caused great attention in empirical studies in economics since Mincer (1974) published the seminal work in 1974. He finds that for white males not working on farms, an extra year of education raises the earnings by about 7%. Earlier studies (Tilak, 1989) show that spread education can substantially reduce inequality within countries. Could et al. (2001) build a model to provide insights into the evolution of wage inequality within and between industries and education groups in the past few decades. Dynamic interdependence between human capital and regional development has also recently caused much attention from researchers in spatial economics.[6] Many studies on external effects of education levels on productivity and wage have been conducted. It is demonstrated by some studies that wages are positively

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[3] *The Solow model is sometimes referred as to the Solow-Swan model because Swan (1956) proposed a model similar to the Solow model. The literature of the neoclassical growth theory is referred to Burmeister and Dobell (1970).*

[4] *Extending neoclassical growth theory to spatial economics is made by, for instance, Richardson (1973), Henderson (1985), and Henderson et al. (1995).*

[5] *For instance, Hanushek and Kimko (2000); Barro (2001); Krueger and Lindahl (2001); and Castelló-Climent and Hidalgo-Cabrillana (2012).*

[6] *See Moretti (2004), Lange and Topel (2006), Henderson (2007), and for review on the literature. It should be noted that Zhang (1991) builds inter-regional growth models with endogenous capital and knowledge in the neoclassical growth with micro-economic foundation. Different from Zhang's previous study which focus on knowledge and research, this study emphasizes human capital and learning through education, economic activities, and creative leisure.*

correlated with local human capital.[7] Some studies, for instance, by Glaeser et al. (1995) and Simon (2004), show that a region's higher productivity will attract more workers and cause the region to grow in population. It is also demonstrated by Berry and Glaeser (2005) and Waldorf (2009) that existing stock of human capital is important for attracting educated in-migrants. As these highly-paid workers move to the region, they compete with existing residents for housing and bid up housing prices (Rauch, 1993). As found by Shapiro (2006), about 40 per cent of the growth effects of human capital are due to increased quality of life. Tselios (2008) studies the relationship between income and educational inequalities in the regions of the European Union, using the European Community Household Panel data survey for 94 regions over the period 1995-2000. The research finding suggests a positive relationship between income and educational inequalities. Winters (2011) studies relations among human capital, education conditions, and quality of life in U.S. metropolitan areas. The study concludes that the quality of life is positively affected by both the local human capital level and the relative importance of higher education institutions (which is measured by the share of the population enrolled in college). Zhu (2011) studies the individual heterogeneity in returns to education in China from 1995-2002. The study provides heterogeneous effects both within and between gender groups. Zhu finds that the heterogeneity in schooling returns falls from 1995 to 2002 for both genders in urban China, although their rates of education return have increased substantially. A reason for the narrowing heterogeneity is due to a more functioning and an increasingly integrated urban labor market in China. All of these studies show the necessity of including endogenous human capital accumulation and education in explaining regional economic development. Our model takes account of human capital and physical capital in a single framework. The physical accumulation in our model is based on the neoclassical growth theory. Our approach to human capital accumulation is influenced by the Uzawa-Lucas two-sector model. Since Romer (1986) re-examined issues of endogenous technological change and economic growth, economists have shown a great interest in endogenous knowledge and economic growth.[8] Specially the work by Lucas (1988) has caused a great interest in formal modeling of education and economic growth among economists. The core model of formal dynamic growth model with education was proposed by Uzawa (1965).[9] In the Uzawa-Lucas model and many of their extensions and generalizations, it is implicitly assumed that all skills and human capital is formed due to formal schooling. Nevertheless, it is a common sense that much of human capital may be accumulated in family and many other social and economic activities. For instance, the human capital of a graduate student from rich family in US may be quite the human capital of a graduate student from a middle class in India. Ignoring nonschool factors may make us misunderstand the role of formal education in economic development. As pointed out by Chen and Chevalier (2008), "Making and exploiting an investment in human capital requires individuals to sacrifice not only consumption, but also leisure. When estimating the returns to education, existing studies typically weigh the monetary costs of schooling (tuition and forgone wages) against increased wages, neglecting the associated labor/leisure tradeoff." Our study is to introduce other sources of learning into the Uzawa-Lucas two-sector growth model.

[7] *See, for instance, Rauch (1993); Dalmazzo and de Blasio (2007a, 2007b).*

[8] *Refer to, for instance, Lucas (1988); Grossman and Helpman (1991); and Aghion and Howitt (1998), for the literature on economic growth and endogenous technological change.*

[9] *Nowadays, the Uzawa model is often called the Uzawa-Lucas model in the literature of economics, except other possible reasons, because Lucas refined and popularized the Uzawa model.*

As pointed out by Glaeser et al. (2001)[10], consumption amenities have increasingly played more important role in economic geography. Public services, local transportation systems, accessibilities, pollution, and human relations such as discrimination all involve externalities and affect amenities. This study incorporates amenity into the consumer location decision by assuming that amenity is an endogenous variable.[11] The regional population dynamics is influenced by many changing characteristics of environmental quality such as levels of noise pollution, air quality, open space, and other physical and social neighborhood qualities at each location. Environmental quality can be reflected in part by its effect on the choice of residential location. Different amenities may exist at any location. Some may be historically given, such as historical buildings and climate; others such as noise and cleanness, may be endogenously determined by the location of residents. People may prefer a low-density residential area to a high one, as there tend to have more green, less noise, more cleanness and more safety in a low-density area. Nevertheless, there are other factors, such as social interactions, which may make high-density area attractive. Many empirical studies are have been conducted to identify the role of amenities on spatial development. Like almost any important issue in economics, different authors provide different (often opposite) conclusions. For instance, Lewis et al. (2002) and Kim et al. (2005) empirically found little significance of natural amenities in influencing regional demographic change. Various reasons are given for explaining the conflicting findings. For instance, the empirical complexity of measuring amenities, partial approaches used in isolating causal relationships, stages of development, and the existence of spatial heterogeneity in the migration effects make it difficult to identify a converging conclusion.[12] As summarized by Storper and Scott (2009), there are three influential ideas about amenities interact with spatial economics: (a) Florida's creative class theory, (b) the research by Glaeser and a number of co-workers which claims that amenities underlie urban growth and (c) Clark's notion of the city as an entertainment machine. According to Florida (2002, 2003, 2004), the presence of a creative class is significant to regional dynamics. Clark claims that amenities (which are specially referred to urban attractions such as parks, art galleries, orchestras, museums and so on) drive urban growth (Clark et al., 2002). Nevertheless, almost all studies on amenity and regional development have not formally dealt with endogenous human and physical capital accumulation. After reviewing the literature about relations among human capital, amenities and spatial economic development, Storper and Scott (2009: 153) state: "... any approach to urban dynamics that is not clearly linked to a basic logic of genesis and early growth must be deficient with regard to its central conceptual capabilities, in the sense that explanation will now be confined to marginal or intermediate adjustments, as opposed to core processes." Our analytical framework can properly deal with the role of amenities in regional economic development as various mechanisms of economic growth are integrated within a consistent framework.

This study is concerned with a small open economy with economic geography.[13] There is a large amount of the literature on growth and capital accumulation of small open

[10]*See, The initial introduction of spatial amenity into utility in a general equilibrium framework was conducted by Zhang (1993) and the initial introduction of spatial amenity into a formal growth model by Zhang (2003). There is a large body of literature on amenities and spatial economics, for instance, equilibrium ideas by Graves (1979) and Roback (1982), turnaround migration theory by Brown et al. (1997), life cycle studies by Clark and Hunter (1992), research on rural development by Deller et al. (2001).*

[11]*Refer to, This study does not take account of externalities for producers. For instance, firms often prefer to locate to other firms. An explicit introduction of externalities will make the spatial structure far more complicated. In the literature of spatial economics, various externalities have been analyzed (see, Henderson, 1974, Upton, 1981, Abdel-Rahman, 2004).*

[12]*For instance, Partridge, et al. (2008), Robbins et al. (2009), and Chi and Marcouiller (2011).*

[13]*Refer to, for instance, Obstfeld and Rogoff (1999), Lane (2001), Kollmann (2001, 2002), Benigno and Benigno (2003), and Galí and Monacelli (2005), for the literature on economics of open economies.*

economies. As far as I know, nevertheless, interregional issues are neglected in almost all of these studies. This is concerned with the determination of goods production, human and physical capital accumulation, and land rent in a multi-regional equilibrium framework. Our model is built upon a few key models in economics – Solow's one-sector growth model, Arrow's learning by doing model, Uzawa-Lucas's growth model with education, the neoclassical growth theory for a small-open economy, and the neoclassical growth trade theory. The synthesis of these theories within a single framework is analytically tractable partly because this study proposes an alternative approach to consumers' behavior. This study integrates the main mechanisms of economic growth in these theories into a single framework. In fact, this study is a synthesis of Zhang's interregional growth model for a small-open economy (Zhang, 2010) and Zhang's two-sector growth model with endogenous human and physical capital (Zhang, 2007). In Zhang's 2007 model, human capital is endogenous and education sector is introduced, but there is no interregional interdependence (as there is a single region). This paper builds a dynamic one-commodity and multiple-region trade model to examine interdependence among international trade, interregional trade and national growth. The rest of the paper is organized as follows. Section 2 builds the model. Section 3 shows that the motion of the economic system is determined by a set of differential equations and simulates the motion of the system with given initial conditions. Section 4 studies effects of changes in some parameters on the economic system. Section 5 concludes the study.

2. The Multi-Region Trade Model with Human and Physical Capital

There is a single good, called industrial good, in the world economy and the price of the industrial good is unity.[14] It is assumed that the national economy is too small to affect the world interest rate. The households hold wealth and receive income from wages and interest payments of wealth. Land is only for residential use. All markets are perfectly competitive and capital and labor are completely mobile between the economy's sectors and capital is perfectly mobile in international market. The possibility of international emigration or/and immigration is neglected. The system consists of multiple regions, indexed by $j = 1, ..., J$. Each region has industrial ad education sectors. Perfect competition is assumed to prevail in good and service markets both within each region and among the regions, and commodities are traded without any barriers such as transport costs or tariffs. Let wage and interest rates be presented by $w_j(t)$ and $r_j(t)$, respectively, in the j th region. The interest rate is identical throughout the national economy, i.e., $r_j(t) = r^*$, where r^* is the rate of interest fixed in the international economy. There is a homogenous population, $\bar{N}$. It is assume that any person lives in the region where he receives education and works. Each region has fixed land, L_j. Land quality, climates, and environment are homogenous within each region, but they vary among regions. The assumption of zero transportation cost of commodities implies price equality for the commodity among regions. As amenity and land are immobile, wage rates and land rent may not be equal among regions.

[14] *The literature on international growth with trade is referred to, for instance, Oniki and Uzawa (1965), Deardorff (1973), Ruffin (1979), Findlay (1984), Eaton (1987), Brecher, et al. (2002), and Sorger (2002). The literature of a small open economy is referred to, for instance, Obstfeld and Rogoff (1998) and Galí and Monacelli (2005).*

Let $H_j(t)$ stand for region j's level of human capital. Let $K(t)$ and $\overline{K}(t)$ represent respectively the capital stocks employed by and the total wealth of the national economy at t. The variables $N_{ij}(t)$ and $K_{ij}(t)$ stand for region j's labor force and capital stocks employed by the production sector, and $N_{ej}(t)$ and $K_{ej}(t)$ for region j's labor force and capital stocks employed by the education sector. Let $T_j(t)$, $T_{ej}(t)$, and $T_{hj}(t)$ stand for, respectively, the work time, study time, and leisure time of a typical worker in region j. The conditions of full employment of labor and capital are

$$K_i(t) + K_e(t) = K_j(t), \quad N_{ij}(t) + N_{ej}(t) = N_j(t), \tag{1}$$

where $K_j(t)$ and $N_j(t)$ stand for respectively the capital used by region j and region j's total qualified labor supply. Here $N_j(t)$ is

$$N_j(t) = T_j(t) H_j^{m_j}(t) \overline{N}_j(t), \tag{2}$$

in which $\overline{N}_j(t)$ is region j's population and m_j is a positive parameter measuring how region j effectively applies human capital. Rewrite (1) as follows

$$n_{ij}(t) k_{ij}(t) + n_{ej}(t) k_{ej}(t) = k_j(t), \quad n_{ij}(t) + n_{ej}(t) = 1, \tag{3}$$

in which

$$k_{qj}(t) \equiv \frac{K_{qj}(t)}{N_j(t)}, \quad n_{qj}(t) \equiv \frac{N_{qj}(t)}{N_j(t)}, \quad k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad q = i, e.$$

Behavior of producers

It is assume that production is to combine 'qualified labor force' $N_{ij}(t)$ and physical capital, $K_{ij}(t)$. The conventional production function is used to describe a relationship between inputs and output. The function $F_{ij}(t)$ stands for the flow of production at time t. The production process is described by

$$F_{ij}(t) = A_{ij} K_{ij}^{\alpha_{ij}}(t) N_{ij}^{\beta_{ij}}(t), \quad A_{ij}, \alpha_{ij}, \beta_{ij} > 0, \quad \alpha_{ij} + \beta_{ij} = 1.$$

where A_{ij}, α_{ij}, and β_{ij} are positive parameters. Markets are competitive; thus labor and capital earn their marginal products. The rate of interest and wage rate are determined by markets. Hence, for any individual firm r^* and $w_j(t)$ are given at each point of time. The production sector chooses the two variables $K_{ij}(t)$ and $N_{ij}(t)$ to maximize its profit. The marginal conditions are

$$r^* + \delta_k = \alpha_{ij} A_{ij} k_{ij}^{-\beta_{ij}}(t), \quad w_j(t) = \beta_{ij} A_{ij} k_{ij}^{\alpha_{ij}}(t), \tag{4}$$

where δ_k is depreciation rate of physical capital.

Education sector

Education sector is treated similarly as the production sector (Zhang, 2007). For simplicity of analysis this study focuses on the case of perfect competition in education. Students pay the education fee $p_j(t)$ per unit of time. Educational service is provided with input factors of human capital and physical capital. The education sector pays teachers and capital with the market rates. The total education service is measured by the total education time received by the population. The production function of the education sector is

$$F_{ej}(t) = A_{ej} K_{ej}^{\alpha_{ej}}(t) N_{ej}^{\beta_{ej}}(t), \ \alpha_{ej}, \beta_{ej} > 0, \ \alpha_{ej} + \beta_{ej} = 1, \tag{5}$$

where A_{ej}, α_{ej} and β_{ej} are positive parameters. The marginal conditions for the education sector are

$$r^* + \delta_k = \alpha_{ej} A_{ej} p_j k_{ej}^{-\beta_{ej}}, \ w_j = \beta_{ej} A_{ej} p_j k_{ej}^{\alpha_{ej}}. \tag{6}$$

Consumer behaviors and wealth dynamics

In order to define incomes, it is necessary to determine land ownership structure. It can be seen that land properties may be distributed in multiple ways under various institutions. To simplify the model, the assumption of "absentee landownership" is accepted. This means that the income of land rent is spent outside the economic system. A possible case is that the land is owned by the government.[15] Households rent the land in competitive market and the government uses the income for military or other public purposes. Consumers make decisions on choice of education, lot size, consumption levels of services and commodities as well as on how much to save. In this study, we follow Zhang (2007) in modeling choice of education time. The preference over current and future consumption is reflected in the consumer's preference structure over education, consumption and saving. Let $\bar{k}_j(t)$ stand for the per capita wealth of region j. Per capita current income from the interest payment $r^* \bar{k}_j(t)$ and the wage payment $H_j^{m_j}(t) T_j(t) w_j(t)$ is given by

$$y_j(t) = r^* \bar{k}_j(t) + H_j^{m_j}(t) T_j(t) w_j(t).$$

The variable $y_j(t)$ is the current income. The sum of money that consumers are using for consuming, saving, and education are not necessarily equal to the current income because consumers can sell wealth to pay, for instance, the current consumption if the current income is not sufficient for consuming. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, it is assumed that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income $\hat{y}_j(t)$ is the sum of the current income and the value of the wealth held. That is

$$\hat{y}_j(t) \equiv y_j(t) + \bar{k}_j(t) = (1 + r^*) \bar{k}_j(t) + T_j(t) \bar{w}_j(t), \tag{7}$$

[15] *Another two popular assumptions in the literature of spatial economics are the equally shared landownership and the public ownership. In the former, land is owned equally by all households in the system. Appendix A2 studies the dynamics for the equally shared landownership. In the latter, for instance as accepted in Kanemoto (1980), the government rents the land from the landowners at certain rent and sublets it to households at the market rent, using the net revenue to subsidize city residents equally.*

where $\overline{w}_j(t) \equiv H_j^{m_j}(t) w_j(t)$. The wage income in region j is $W_j(t) = \overline{w}_j(t) T_j(t)$. The disposable income is used for saving, consumption, and education. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p_0(t)\bar{k}_j(t)$ with $p_0(t) = 1$), is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, $\bar{k}_j(t)$ may be considered as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute among saving, consumption and education. In the growth literature, for instance, in the Solow model, the saving is out of the current income, $y_j(t)$, while in this study the saving is out of the disposable income, $\hat{y}_j(t)$.

The consumer is faced with the following time constraint

$$T_j(t) + T_{ej}(t) + T_{hj}(t) = T_0, \tag{8}$$

where T_0 is the total available time for work, study and leisure.[16] At each point of time, a consumer would distribute the total available budget among saving $s_j(t)$, housing (measured in lot size) $l_j(t)$, consumption of goods $c_j(t)$, and education $T_{ej}(t)$. The budget constraint is given by

$$R_j(t) l_j(t) + c_j(t) + s_j(t) + p_j(t) T_{ej}(t) = \hat{y}_j(t) = \left(1 + r^*\right) \bar{k}_j(t) + T_j(t) \overline{w}_j(t). \tag{9}$$

where $R_j(t)$ is the land rent in region j. Equation (9) means that consumption, housing, education and savings exhaust the consumers' disposable income.

Substituting (8) into (9) yields

$$R_j(t) l_j(t) + c_j(t) + s_j(t) + \bar{p}_j(t) T_{ej}(t) + \overline{w}_j(t) T_{hj}(t) = \bar{y}_j(t) \equiv \left(1 + r^*\right) \bar{k}_j(t) + T_0 \, \overline{w}_j(t), \tag{10}$$

where $\bar{p}_j(t) \equiv p_j(t) + \overline{w}_j(t)$. The new variables $\bar{p}_j(t)$ and $\overline{w}_j(t)$ are respectively the opportunity costs of education and leisure.

It is assumed that consumers' utility function is a function of level of goods $c_j(t)$, lot size $l_j(t)$, level of saving $s_j(t)$, education service $T_{ej}(t)$, and leisure $T_{hj}(t)$ as follows

$$U_j(t) = u_j(t) l_j^{\gamma_0}(t) c_j^{\xi_0}(t) s_j^{\lambda_0}(t) T_{ej}^{\eta_0}(t) T_{hj}^{\sigma_0}(t), \quad \gamma_0, \xi_0, \lambda_0, \eta_0, \sigma_0 > 0, \tag{11}$$

where γ_0 is called the propensity to consume housing, ξ_0 the propensity to consume the commodity, λ_0 the propensity to own wealth, η_0 the propensity to receive education, and σ_0 the propensity to consume leisure. This utility function is proposed first by the early 1990s by Zhang as an alternative approach to household behavior and is applied to different economic problems (Zhang, 2005).

[16] *Another As one referee points out, the household's childbearing decision depends crucially on economic conditions. The available time should be divided the time labor effort, education, childrearing and leisure. As introduction of endogenous population further complicate the already labyrinthine model, for simplicity we omit issues related to endogenous population.*

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Here, education has two kinds of returns. As education raises labor productivity, its effect is reflected in higher wages. As Lazear (1977: 570) describes: "education is simply a normal consumption good and that, like all other normal goods, an increase in wealth will produce an increase in the amount of schooling purchased. Increased incomes are associated with higher schooling attainment as the simple result of an income effect." Education also brings about direct pleasure, more knowledgeable, higher social status and so on.[17] The relative importance of these returns may vary across different types of education with different individuals. It should be noted that in a recent study by Chen (2012), it is shown that regional economic and social conditions have strong impact on anadolescent's educational achievement. This study may take account of these factors by the utility function (which determines which region the household will choose to reside) and human capital accumulation equation (to be specified late on).

In (11), $u_j(t)$ is called region j's amenity level. The concept of amenity measures a region's attractiveness for households. Amenities are affected by infrastructures, regional cultures and climates.[18] People cluster together for different reasons. For instance, people like to socially interact with each other. This implies that a large population of the region is attractive. A region with large population offers more opportunities for employment. Good climates make a region attractive as residential location. This study assumes that amenity is affected by production and consumption activities. The amenity u_j is specified

$$u_j(t) = \bar{\theta}_j F_{ij}^{a_i}(t) F_{ej}^{a_e}(t) \bar{N}_j^b(t), \quad j = 1, \cdots, J, \tag{12}$$

where $\bar{\theta}_j \ (> 0)$, a_i, a_e, and b are parameters. The signs of a_i, a_e, and b are not specified as economic activities and population may have either positive or negative effects on regional attractiveness. In fact, there are many factors that may affect amenities. For instance, an improvement in highway networks and airport facilities increases regional amenities (Rasker *et al.*, 2009). According to Chi and Marcouiller (2011), natural amenities are strongly affected by economic conditions.

The consumer maximizes $U_j(t)$ subject to (9). The first-order conditions are

$$R_j(t) l_j(t) = \gamma \bar{y}_j(t), \ c_j(t) = \xi \bar{y}_j(t), \ s_j(t) = \lambda \bar{y}_j(t), \ \bar{p}_j(t) T_{ej}(t) = \eta \bar{y}_j(t), \ \bar{w}_j(t) T_{hj}(t) = \sigma \bar{y}_j(t), \tag{13}$$

where

$$\gamma \equiv \rho \gamma_0, \ \xi \equiv \rho \xi_0, \ \lambda \equiv \rho \lambda_0, \ \eta \equiv \rho \eta_0, \ \sigma \equiv \rho \sigma_0, \ \rho \equiv \frac{1}{\gamma_0 + \xi_0 + \lambda_0 + \eta_0 + \sigma_0}.$$

[17] *See, for instance, Heckman (1976); Lazear (1977); and Malchow-Møller, et al. (2011).*
[18] *See, for instance, Kanemoto (1980), Diamond and Tolley (1981), Blomquist et al (1988), Andersson et al. (2003).*

The demand for education is given by $T_{ej} = \eta \bar{y}_j / \bar{p}_j$. The demand for education decreases in the price of education and the wage rate and increases in $\bar{y}_j$. An increase in the propensity to receive education increases the education time when the other conditions are fixed.

As the homogeneous households are assumed to be freely mobile among the regions, the utility level of people should be equal, irrespective of in which region they live, i.e.

$$U_j(t) = U_m(t), \quad j, m = 1, \cdots, J. \tag{14}$$

This assumption agrees with the idea that people tend to move to places where their utility can be maximized (Tiebout, 1956). Here possible costs for migration are neglected. As argued by Adam Smith in *Wealth of Nations*, "it appears evidently from experience that a man is of all sorts of luggage the most difficult to be transported." (David *et al*., 2010). Although it is conceptually not difficult to introduce migration costs into the model, it will become difficult to analyze the model. In this study, instead of wage equalization (which is often used as the equilibrium mechanism of population distribution), it is assumed that consumers obtain the same level of utility in different regions as the equilibrium mechanism of population distribution among the regions. Although the condition of utility equalization is often used in the literature of urban economics,[19] the assumption of utility equalization is rarely used in the literature of economic dynamics as the temporary equilibrium condition of population distribution. It is argued that this assumption is more reasonable than the assumption of wage equalization in interregional analysis.[20]

According to the definitions of $s_j(t)$, the wealth accumulation of the representative household in region j is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{15}$$

This equation simply states that the change in wealth is equal to the savings minus dissavings.

Dynamics of human capital

People may accumulate human capital through different ways. Different forms of learning have different effects of human capital accumulation (Aakvik, *et al.,* 2010). For instance, social and economic conditions of the family and school quality are crucial to educational achievement.[21] Neighbourhood characteristics may also have strong effects on human capital accumulation.[22] It is assumed that there are three sources of improving human capital, through education, "learning by producing", and "learning by leisure". Arrow (1962) first introduced learning by doing into growth theory; Uzawa (1965) took account of trade-offs between investment in education and capital accumulation; and Zhang (2007) introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory. Following Zhang (2007), human capital dynamics is given by

[19] *Fujita (1989) provides some models with the assumption of equalizing utility levels among households in different locations.*

[20] *The importance of social capital such as local ties in determining mobility of people has been well recognized in the literature (e.g., Jacob, 1961; Schiff, 1992; Glaeser et al., 2002). In a recent study, David et al. (2010) use the concept of social capital mobility to explain crosscountry variations in geographical mobility in Europe, and in particular why it is lower in the South and higher in the North of Europe.*

[21] *Kane (1994); Maurin (2002); and Dearden et al. (2002).*

[22] *Haveman et al. (1991); Borjas (1995); Harding (2003); and Andersson and Subramanian (2006).*

$$\dot{H}_j(t) = \frac{\upsilon_{ej}\, F_{ej}^{a_{ej}}(t)\left(H_j^{m_j}(t)T_{ej}(t)\bar{N}_j(t)\right)^{b_{je}}}{H_j^{\pi_{ej}}(t)\bar{N}_j(t)} + \frac{\upsilon_{ij}\, F_{ij}^{a_{ij}}(t)}{H_j^{\pi_{ij}}(t)\bar{N}_j(t)} + \frac{\upsilon_{hj}\, C_j^{a_{hj}}(t)T_{hj}^{b_{hj}}(t)}{H_j^{\pi_{hj}}(t)\bar{N}_j(t)} - \delta_{hj}\, H_j(t), \quad (16)$$

where $\delta_{hj}\ (> 0)$ is the depreciation rate of human capital, υ_{ej}, υ_{ij}, υ_{hj}, a_{ej}, b_{ej}, a_{ij}, and a_{hj} are non-negative parameters. The signs of the parameters π_{ej}, π_{ij}, and π_{hj} are not specified as they may be either negative or positive.

The above equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation.[23] The term

$$\frac{\upsilon_{ej}\, F_{ej}^{a_{ej}}(t)\left(H_j^{m_j}(t)T_{ej}(t)\bar{N}_j(t)\right)^{b_{ej}}}{H_j^{\pi_{ej}}(t)\bar{N}_j(t)}$$

describes the contribution to human capital improvement through education. Human capital tends to increase with an increase in the level of education service, F_{ej}, and in the (qualified) total study time, $H_j^{m_j}\, T_{ej}\, \bar{N}_j$. Here, students' human capital and efforts affect their human capital accumulation. In traditional approach like in the Uzawa-Lucas model, the result of education is only determined by input of the education sector. This study, considers students' "qualified effort" also important in the human capital accumulation. The population $\bar{N}_j$ in the denominator measures the contribution in terms of per capita. The term $H^{\pi_{ej}}$ indicates that as the level of human capital of the population increases, it may be more difficult (in the case of π_{ej} being positive) or easier (in the case of π_{ej} being negative) to accumulate more human capital via formal education. This model takes account of learning by producing effects in human capital accumulation by the term $\upsilon_{ij}\, F_i^{a_{ij}} / H_j^{\pi_{ij}}$. This term implies that contribution of the production sector to human capital improvement is positively related to its production scale F_{ij} and is dependent on the level of human capital. The term $H^{\pi_{ij}}$ takes account of returns to scale effects in human capital accumulation. The case of $\pi_{ij} > (<)\ 0$ implies that as human capital is increased it is more difficult (easier) to further improve the level of human capital. The term $\upsilon_{hj}\, C_j^{a_{hj}} / H_j^{\pi_{hj}}\, \bar{N}_j$ accounts for learning by consuming. This term can be interpreted similarly as the term for learning by producing.

It should be noted that in the literature on education and economic growth, it is assumed that human capital evolves according to the following equation (see Barro and Sala-i-Martin, 1995)

$$\dot{H}(t) = H^{\eta}(t)G(T_e(t)),$$

where the function G is increasing as the effort rises with $G(0) = 0$. This formation is due to Lucas (1988). Uzawa's model may be considered a special case of the Lucas model with $\gamma = 0$, $U(c) = c$, and the assumption that the right-hand side of the above equation is linear in the effort. It seems reasonable to consider diminishing returns in human capital accumulation. For instance, people accumulate human capital rapidly early in life, then less rapidly, then not at all – as though each additional percentage increment were harder to gain than the preceding one. Solow adapts the Uzawa formation to the following form

[23] *A detailed explanation is referred to Zhang (2007).*

$$\dot{H}(t) = H(t)\kappa T_e(t).$$

The new formation implies that if no effort is devoted to human capital accumulation, then $\dot{H}(0) = 0$ (human capital does not vary as time passes; this results from depreciation of human capital being ignored); if all effort is devoted to human capital accumulation, then $g_H(t) = \kappa$ (human capital grows at its maximum rate; this results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$.

Demand for and supply of education and trade balances

For the education sector, the demand and supply balances at any point of time

$$T_{ej}(t)\bar{N}_j(t) = F_{ej}(t). \tag{17}$$

The total capital stock used by the economy is the sum of the capital stocks employed by the capital stocks employed by all the regions. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) \; = \sum_{j=1}^{J}\left[k_{ij}(t)N_{ij}(t) + k_{ej}(t)N_{ej}(t)\right]. \tag{18}$$

The total wealth of the national economy is the sum of the wealth owned by all the households

$$\bar{K}(t) = \sum_{j=1}^{J}\bar{k}_j(t)N_j(t). \tag{19}$$

The variable $B(t)$ stands for the value of the economy's net foreign assets at t. The income from the net foreign assets, $X_0(t)$, which may be either positive, zero, or negative, is equal to $r^*B(t)$. The national industrial output is equal to the national net saving. That is

$$S(t) + C(t) - \bar{K}(t) - r^*B(t) + \sum_{j=1}^{J}\delta_{kj}K_j(t) = F(t), \tag{20}$$

where

$$C(t) \equiv \sum_{j=1}^{J} c_j(t)\bar{N}_j(t), \;\; S(t) \equiv \sum_{j=1}^{J} s_j(t)\bar{N}_j(t), \;\; F(t) \equiv \sum_{j=1}^{J} F_{ij}(t).$$

The assumption that labor force and land are fully employed is represented by

$$\sum_{j=1}^{J}\bar{N}_j(t) = \bar{N}, \;\; l_j(t)\bar{N}_j(t) = L_j, \;\; j = 1, \cdots, J. \tag{21}$$

The multi-regional model of a small open economy with physical and human capital accumulation is thus completed. The rest of the study examines behavior of the system.

3. The Dynamics of the National Economy

Multi-regional dynamics system is complicated. Nevertheless, it is shown that its motion is given by a set of differential equations. The following lemma shows how to determine the motion of all the variables in the dynamic system.

Lemma 1

The variables, k_{ij}, w_j, k_{ej}, and p_j are uniquely determined as functions of r^*. The motion of the levels of the per-capita wealth and human capital is given by

$$\begin{aligned}
&\dot{\bar{k}}_j(t) = \Lambda_j\left(\left(\bar{k}_j(t)\right), \left(H_j(t)\right)\right), \\
&\dot{H}_j(t) = \Psi_j\left(\left(\bar{k}_j(t)\right), \left(H_j(t)\right)\right),
\end{aligned} \tag{22}$$

in which $\Lambda_j\left(\left(\bar{k}_j\right), \left(H_j\right)\right)$ and $\Psi_j\left(\left(\bar{k}_j\right), \left(H_j\right)\right)$ are functions of $\left(\bar{k}_j\right)$ and $\left(H_j\right)$ defined in the Appendix. The other variables are uniquely determined as functions of $\left(\bar{k}_j\right)$ and $\left(H_j\right)$ at any point of time by the following procedure: $\bar{N}_j$ by (A13) and (A14) → k_{ij} and w_j by (A1) → p_j and k_{ej} by (A2) → F_{ij} and F_{ej} by (A10) → $\bar{y}_j$ by (10) → c_j, s_j, T_{ij} and T_{ej} by (13) → $l_j = L_j / \bar{N}_j$ → $R_j = \xi \bar{y}_j / l_j$ → T_j by (A6) → $N_j = T_j H_j^{m_j} \bar{N}_j$ → k_j by (A8) → $C_j = c_j \bar{N}_j$ → n_{ij} and n_{ej} by (A3) → $N_{ij} = n_{ij} N_j$ and $N_{ej} = n_{ej} N_j$ → $K_{ij} = k_{ij} N_{ij}$ and $K_{ej} = k_{ej} N_{ej}$ → K by (A18) → $\bar{K}$ by (19) → U_j by (A11).

The dynamic equations for the economy are given by the lemma. The system is nonlinear and is of high dimension. It is difficult to generally analyze behavior of the system. To illustrate motion of the system, the parameter values should be specified first. The rate of interest and the total population are specified as follows

$$r^* = 0.04, \ \bar{N} = 10, \ \delta_k = 0.05, \ m_j = 0.8, \ T_0 = 1.$$

The rate of interest is percent and the population is All the regions have the same efficiency of human capital utilization. The physical capital depreciation rate is equal among the regions. The total available time is unity. The parameters in the regions' production functions are specified as follows

$$A_{i1} = 1.2, \ A_{i2} = 1.1, \ A_{i3} = 1, \ \alpha_{i1} = \alpha_{i3} = 0.32, \ \alpha_{i2} = 0.35, \ A_{e1} = 1, \ A_{e2} = 0.9, \ A_{e3} = 0.8,$$
$$\alpha_{e1} = \alpha_{e3} = 0.45, \ \alpha_{e2} = 0.4.$$

Region total productivities in the two sectors are highest, while region total productivities in the two sectors are lowest. Region is called as the coastal region (CR), region the inner region (IR), and region 3 the hinterland region (HR). It should be remarked that although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. For instance, some empirical studies on the US economy demonstrate that the value of the parameter, α, in the Cobb-Douglas production is approximately equal to (for instance, Abel and Bernanke, 1998). With regard to the technological parameters, what are important in our interregional study are their relative values. This is similarly true for the specified differences in land and amenity parameters among regions. The regions' land areas and amenity parameters are

$$L_1 = 4,\ L_2 = 4,\ L_3 = 6,\ \theta_1 = 4,\ \theta_2 = 3.8,\ \theta_3 = 4.3,\ a_i = 0.1,\ a_e = 0.2,\ b = -0.1.$$

The HR has the largest residential area. Here, it is assumed that both the production scales of the two sectors have positive impact on a region's amenity and the education sector's scale effect is stronger than the industrial sector. Moreover, the residential population has a negative effect on the region's amenity. The household preference is specified as follows

$$\lambda_0 = 0.7,\ \gamma_0 = 0.06,\ \xi_0 = 0.1,\ \sigma_0 = 0.2,\ \eta_0 = 0.05.$$

The propensity to consume the goods is higher than the propensity to consume housing, while the propensity to consume housing is higher than to receive education. The parameters in the three regions' human capital accumulation equations are specified as follows

$$\begin{gathered} v_{e1} = 1,\ v_{i1} = 2.5,\ v_{h1} = 1,\ v_{e2} = 0.9,\ v_{i2} = 2.5,\ v_{h2} = 0.8,\ v_{e3} = 0.8,\ v_{i3} = 2.5,\ v_{h3} = 0.7, \\ a_{ej} = 0.3,\ b_{ej} = 0.5,\ a_{ij} = 0.4,\ a_{hj} = 0.1,\ b_{hj} = 0.3,\ \pi_{ej} = 0.1,\ \pi_{ij} = 0.7,\ \pi_{hj} = 0.1. \end{gathered}$$

The depreciation rate of human capital is equal among the three regions, $\delta_{hj} = 0.05$. The function for learning by doing is the same among the three regions. The CR is most effective in education and learning through consuming; while the HR is least effective. With the specified parameter values, we first calculate the time-independent variables

$$\begin{gathered} p_1 = 0.896,\ p_2 = 1.087,\ p_3 = 0.946,\ k_{i1} = 8.45,\ k_{i2} = 9.36,\ k_{i3} = 6.46, \\ k_{e1} = 14.68,\ k_{e2} = 11.58,\ k_{e3} = 11.23,\ w_1 = 1.62,\ w_2 = 1.56,\ w_3 = 1.24. \end{gathered} \tag{24}$$

The price of education in the CR is lower than the other two regions, while the price in the HR is lower than the price in the IR. The wage rate of the CR is higher than the other two regions, while the HR has the lowest wage rate. The equilibrium values of human capital, physical capital, and total labor input of regions are listed as follows

$$\begin{gathered} H_1 = 14.74,\ H_2 = 16.91,\ H_3 = 18.06,\ N_1 = 13.77,\ N_2 = 10.59,\ N_3 = 8.47,\ K_1 = 116.55, \\ K_2 = 99.15,\ K_3 = 54.86,\ F_{i1} = 27.19,\ F_{i2} = 23.08,\ F_{i3} = 15.33,\ F_{e1} = 0.13,\ F_{e2} = 0.097, \\ F_{e3} = 0.095,\ n_{e1} = 0.0027,\ n_{e2} = 0.0034,\ n_{e3} = 0.0038,\ X_0 = -0.62. \end{gathered}$$

In equilibrium the HR has highest level of human capital and highest ratio of labor force located to education. The labor force of the CR is highest among the three regions. Also the CR uses much more physical capital than the other two regions. The HR locates more labor force to education than the other two regions. The country is in trade deficit. The population distribution, the levels of per capita GDP and the amenity levels are given as follows

$$\begin{gathered} \bar{N}_1 = 4.53,\ \bar{N}_2 = 3.11,\ \bar{N}_3 = 2.36,\ g_1 = 6.03,\ g_2 = 7.45,\ g_3 = 6.54,\ u_1 = 3.17, \\ u_2 = 2.91,\ u_3 = 3.24, \end{gathered}$$

where $g_j \equiv \left(F_{ij} + p_{ej} F_{ej}\right)/\bar{N}_j$ is region $j's$ per capita GDP. The CR has the largest population, while the HR has the smallest population. The CR has the highest amenity level and the HR has the lowest amenity level. The time distributions, consumption levels and levels of per capita wealth are as follows

$T_1 = 0.353$, $T_2 = 0.354$, $T_3 = 0.354$, $T_{h1} = 0.524$, $T_{h2} = 0.524$, $T_{h3} = 0.524$, $T_{e1} = 0.123$, $T_{e2} = 0.122$, $T_{e3} = 0.121$, $c_1 = 3.64$, $c_2 = 3.92$, $c_3 = 3.27$, $\bar{k}_1 = 25.47$, $\bar{k}_2 = 25.53$, $\bar{k}_3 = 22.91$.

The time allocations among the regions are only slight. The household in the IR consumes more than the other two regions, while the consumption level of the household in the HR is lowest. The regional lot sizes and land rents are given as follows

$$l_1 = 0.29,\; l_2 = 0.38,\; l_3 = 0.71,\; R_1 = 7.52,\; R_2 = 6.25,\; R_3 = 2.77.$$

The household in the CR lives in a small house and pays high rent. The household in the HR lives in a large house and pays low rent.

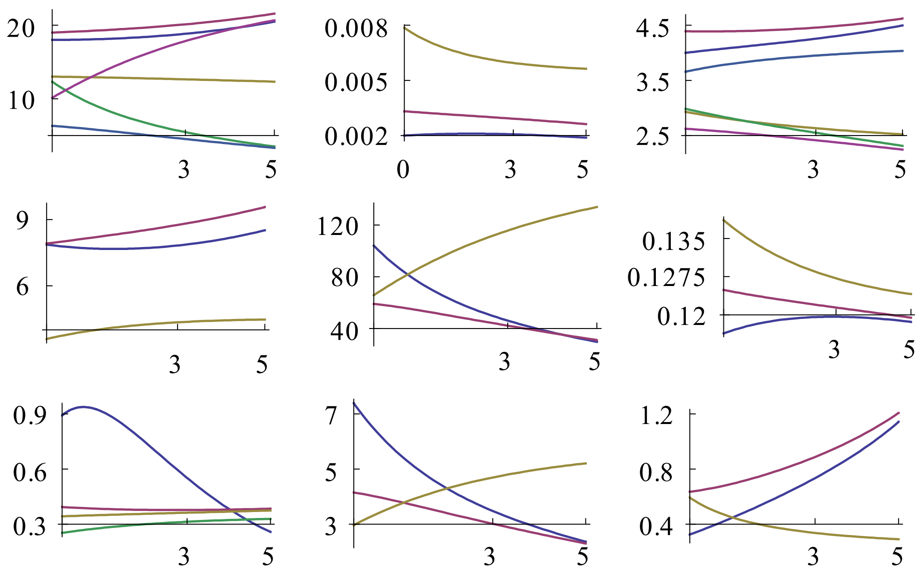
In order to determine stability of the equilibrium point, we calculate the six eigenvalues as follows

$$\{-0.468, -0.433, -0.341, 0.104, 0.073, -0.051\}.$$

Hence, the equilibrium point is a saddle point. This implies that the economic system "normally" will experience economic crisis or unlimited growth in the long term. It is now shown how the system changes over a short period of time for the following initial conditions

$$\bar{k}_1(0) = 27,\; \bar{k}_2(0) = 31,\; \bar{k}_3(0) = 22,\; H_1(0) = 18,\; H_2(0) = 19,\; H_3(0) = 13.$$

The human capital levels in the CR and IR rise over time, but the human capital in the HR falls. The labor and capital inputs in CR and IR fall, but the labor and capital inputs in the HR rise. The consumption levels and lot sizes in the CR and IR are increased, while the corresponding variables in the HR are reduced. Some households move to the HR from the other two regions. The land rent in the HR is increased, while the land rents in the other two regions are reduced. The amenity is improved in the HR, while the residential environment is deteriorated in the other two regions. The time spent on education in the CR and HR falls, while the time in the IR rises initially and falls late on.



4. Comparative Dynamic Analysis

The previous section plots the motion of the variables. This section examines how changes in some parameters affect the national economy. As the model is explicitly solved, it is straightforward to make comparative dynamic analysis.

The effects of a rise on the rate of interest

This section studies what will happen to the equilibrium of the economic system if the rate of interest is changed as follows[24]: $r^* = 0.04 \Rightarrow 0.05$. The rate rises from 4% to 5% in the international market. Let $\bar{\Delta}x(t)$ stand for the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value. The effects on k_j, f_j and w_j are given as follows

$$\bar{\Delta} p_1 = 2.03,\ \bar{\Delta} p_2 = 0.81,\ \bar{\Delta} p_3 = 2.03,\ \bar{\Delta} k_{i1} = -14.35,\ \bar{\Delta} k_{i2} = -14.96,$$
$$\bar{\Delta} k_{i3} = -14.35,\ \bar{\Delta} k_{e1} = -14.35,\ \bar{\Delta} k_{e2} = -14.96,\ \bar{\Delta} k_{e3} = -14.35,$$
$$\bar{\Delta} w_1 = -4.84,\ \bar{\Delta} w_2 = -5.52,\ \bar{\Delta} w_3 = -4.84.$$

The prices of education in the three regions are increased as the cost of capital is increased. It should be noted that the price in the IR is changed less than the prices in the other two regions. The capital stocks by per labor force and wage rates are reduced. The wage rate in the IR is reduced less than in the other two regions. The effects on the human capital, physical capital, and total labor input of regions are

$$\bar{\Delta} H_1 = -0.92,\ \bar{\Delta} H_2 = 0.08,\ \bar{\Delta} H_3 = -0.76,\ \bar{\Delta} N_1 = -3.11,\ \bar{\Delta} N_2 = -3.55,\ \bar{\Delta} N_3 = -3.63,$$
$$\bar{\Delta} K_1 = -16.98,\ \bar{\Delta} K_2 = -18.81,\ \bar{\Delta} K_3 = -17.41,\ \bar{\Delta} F_{i1} = -7.86,\ \bar{\Delta} F_{i2} = -9.88,\ \bar{\Delta} F_{i3} = -8.37,$$
$$\bar{\Delta} F_{e1} = 11.37,\ \bar{\Delta} F_{e2} = 10.2,\ \bar{\Delta} F_{e3} = 10.50,\ \bar{\Delta} n_{e1} = 23.25,\ \bar{\Delta} n_{e2} = 23.18,\ \bar{\Delta} n_{e3} = 22.94,$$
$$\bar{\Delta} X_0 = -284.41.$$

The labor and capital inputs in the three regions are reduced. The human capital levels in the CR and HR are reduced, but the human capital level in the IR is increased. The output levels of the industrial sector in the three regions are reduced, but the output levels of the education sector in the three regions are increased. The trade balance is deteriorated. The population distribution, the levels of per capita GDP and the amenity levels are given as follows

$$\bar{\Delta} \bar{N}_1 = 0.88,\ \bar{\Delta} \bar{N}_2 = -1.41,\ \bar{\Delta} \bar{N}_3 = 0.17,\ \bar{\Delta} g_1 = -8.58,\ \bar{\Delta} g_2 = -8.50,\ \bar{\Delta} g_3 = -8.40,$$
$$\bar{\Delta} u_1 = 1.26,\ \bar{\Delta} u_2 = 1.05,\ \bar{\Delta} u_3 = 1.11.$$

Some households migrate to the CR and HR from the IR. The levels of the per capita GDP in the three regions are reduced, while the levels of amenity in the three regions are increased. The changes in the time distributions, consumption levels and levels of per capita wealth are

[24] *The dynamics is explicitly solved, it is possible to carry out comparative dynamic analysis by assuming that the rate of interest varies in time, r*(t)*

$$\bar{\Delta} T_1 = -3.25,\ \bar{\Delta} T_2 = -3.24,\ \bar{\Delta} T_3 = -3.20,\ \bar{\Delta} T_{h1} = \bar{\Delta} T_{h2} = \bar{\Delta} T_{h3} = 1.87,\ \bar{\Delta} T_{e1} = 1.39,$$
$$\bar{\Delta} T_{e2} = 1.41,\ \bar{\Delta} T_{e3} = 1.31,\ \bar{\Delta} c_1 = -3.77,\ \bar{\Delta} c_2 = -3.69,\ \bar{\Delta} c_3 = -3.65,\ \bar{\Delta} \bar{k}_1 = -3.77,$$
$$\bar{\Delta} \bar{k}_2 = -3.69,\ \bar{\Delta} \bar{k}_3 = -3.65.$$

The work time is reduced, while the time spent on leisure and education are increased. The consumption levels and levels of per capita wealth are reduced. The changes in the lot sizes and levels of the land rent are

$$\bar{\Delta} l_1 = 3.21,\ \bar{\Delta} l_2 = -4.76,\ \bar{\Delta} l_3 = 3.76,\ \bar{\Delta} R_1 = -6.77,\ \bar{\Delta} R_2 = -8.07,\ \bar{\Delta} R_3 = -7.14.$$

The impact of a rise in the propensity to receive education

Let the propensity to receive education be changed in the following way: $\eta_0 : 0.05 \Rightarrow 0.07$. The parameter change has no impact on the following variables

$$\bar{\Delta} p_j = \bar{\Delta} k_{ij} = \bar{\Delta} k_{ej} = \bar{\Delta} w_j = 0.$$

The effects on the human capital, physical capital, and total labor input of regions are as follows

$$\bar{\Delta} H_1 = 5.02,\ \bar{\Delta} H_2 = 5.46,\ \bar{\Delta} H_3 = 5.58,\ \bar{\Delta} N_1 = 0.90,\ \bar{\Delta} N_2 = -0.84,\ \bar{\Delta} N_3 = -1.29,$$
$$\bar{\Delta} K_1 = 0.96,\ \bar{\Delta} K_2 = -0.81,\ \bar{\Delta} K_3 = -1.20,\ \bar{\Delta} F_{i1} = 0.80,\ \bar{\Delta} F_{i2} = -0.95,\ \bar{\Delta} F_{i3} = -1.41,$$
$$\bar{\Delta} F_{e1} = 35.10,\ \bar{\Delta} F_{e2} = 32.28,\ \bar{\Delta} F_{e3} = 31.55,\ \bar{\Delta} n_{e1} = 33.90,\ \bar{\Delta} n_{e2} = 33.40,\ \bar{\Delta} n_{e3} = 33.26,$$
$$\bar{\Delta} X_0 = 13.72.$$

As the propensity to receive education is increased, the levels of human capital in the three regions are increased. The labor force and capital stocks employed by the CR is increased, while the levels of labor force and capital stocks employed by the IR and the HR are reduced. The output levels of the education sector in the three regions are increased. The output level of the industrial sector in the CR is increased, while the output levels of the industrial sector in the other two sectors are reduced. The trade balance is deteriorated. From the following results, it is concluded that some households migrate from the IR and the HR to the CR. The CR's per capita GDP falls, while the other two regions' per capital GDP rises. The amenity levels in the three regions are increased.

$$\bar{\Delta} \bar{N}_1 = 1.32,\ \bar{\Delta} \bar{N}_2 = -0.85,\ \bar{\Delta} \bar{N}_3 = -1.42,\ \bar{\Delta} g_1 = -0.37,\ \bar{\Delta} g_2 = 0.04,\ \bar{\Delta} g_3 = 0.20,$$
$$\bar{\Delta} u_1 = 6.15,\ \bar{\Delta} u_2 = 5.74,\ \bar{\Delta} u_3 = 5.64.$$

The changes in the time distributions, consumption levels and levels of per capita wealth are

$$\bar{\Delta} T_1 = -4.25,\ \bar{\Delta} T_2 = -4.16,\ \bar{\Delta} T_3 = -4.13,\ \bar{\Delta} T_{h1} = \bar{\Delta} T_{h2} = \bar{\Delta} T_{h3} = -4.98,\ \bar{\Delta} T_{e1} = 33.34,$$
$$\bar{\Delta} T_{e2} = 33.41,\ \bar{\Delta} T_{e3} = 33.43,\ \bar{\Delta} c_1 = -1.17,\ \bar{\Delta} c_2 = -0.85,\ \bar{\Delta} c_3 = -0.76,\ \bar{\Delta} \bar{k}_1 = -1.17,$$
$$\bar{\Delta} \bar{k}_2 = -0.85,\ \bar{\Delta} \bar{k}_3 = -0.76.$$

The education time of each household is increased, while the leisure and work time, consumption level and wealth of each household is reduced. The effects on the lot sizes and land rent are listed as follows

$$\overline{\Delta} l_1 = -0.89, \ \overline{\Delta} l_2 = 0.85, \ \overline{\Delta} l_3 = 1.31, \ \overline{\Delta} R_1 = -0.29, \ \overline{\Delta} R_2 = -1.68, \ \overline{\Delta} R_3 = -2.04.$$

Although the CR's population is increased, the land rent in the CR is reduced. A rise in the population tends to increase the land rent. On the other hand, the reduction in the work time reduces the income. The balance between the two opposite forces results in the fall in the land rent.

The impact of a rise in the propensity to save

We now change the propensity to save in the following way: $\lambda_0 : 0.70 \Rightarrow 0.72$. The parameter change has no impact on the following variables

$$\overline{\Delta} p_j = \overline{\Delta} k_{ij} = \overline{\Delta} k_{ej} = \overline{\Delta} w_1 = 0.$$

The effects on the human capital, physical capital, and total labor input of regions are as follows

$$\overline{\Delta} H_1 = -0.02, \ \overline{\Delta} H_2 = -0.34, \ \overline{\Delta} H_3 = 0.41, \ \overline{\Delta} N_1 = -0.39, \ \overline{\Delta} N_2 = -0.06,$$
$$\overline{\Delta} N_3 = -0.86, \ \overline{\Delta} K_1 = -0.38, \ \overline{\Delta} K_2 = -0.06, \ \overline{\Delta} K_3 = -0.86, \ \overline{\Delta} F_{i1} = -0.39,$$
$$\overline{\Delta} F_{i2} = -0.06, \ \overline{\Delta} F_{i3} = -0.87, \ \overline{\Delta} F_{e1} = 0.22, \ \overline{\Delta} F_{e2} = 0.78, \ \overline{\Delta} F_{e3} = -0.57,$$
$$\overline{\Delta} n_{e1} = 0.61, \ \overline{\Delta} n_{e2} = 0.84, \ \overline{\Delta} n_{e3} = 0.29, \ \overline{\Delta} X_0 = -56.90.$$

The levels of human capital in the CR and the IR are reduced, while the level of human capital in the HR is increased. The labor force and capital inputs and levels of the industrial sector are reduced. The output levels of the education sector in the CR and the IR are reduced, while the output level of the education sector in the HR is increased. The trade balance is improved. The population distribution, the levels of per capita GDP and the amenity levels are given as follows

$$\overline{\Delta} \bar{N}_1 = 0.02, \ \overline{\Delta} \bar{N}_2 = 0.59, \ \overline{\Delta} \bar{N}_3 = -0.81, \ \overline{\Delta} g_1 = -0.40, \ \overline{\Delta} g_2 = -0.64, \ \overline{\Delta} g_3 = -0.06,$$
$$\overline{\Delta} u_1 = 0.004, \ \overline{\Delta} u_2 = 0.09, \ \overline{\Delta} u_3 = -0.12.$$

The changes in the time distributions, consumption levels and levels of per capita wealth are

$$\overline{\Delta} T_1 = -0.38, \ \overline{\Delta} T_2 = -0.39, \ \overline{\Delta} T_3 = 0.21, \ \overline{\Delta} T_{h1} = \overline{\Delta} T_{h2} = \overline{\Delta} T_{h3} = 0.21, \ \overline{\Delta} T_{e1} = 0.21, \ \overline{\Delta} T_{e2} = 0.19,$$
$$\overline{\Delta} T_{e3} = 0.23, \ \overline{\Delta} c_1 = 0.19, \ \overline{\Delta} c_2 = 0.22, \ \overline{\Delta} c_3 = 0.41, \ \overline{\Delta} \bar{k}_1 = 3.05, \ \overline{\Delta} \bar{k}_2 = 2.80, \ \overline{\Delta} \bar{k}_3 = 3.42.$$

The effects on the lot sizes and land rent are listed as follows

$$\overline{\Delta} l_1 = -0.39, \ \overline{\Delta} l_2 = -0.06, \ \overline{\Delta} l_3 = 0.87, \ \overline{\Delta} R_1 = -0.19, \ \overline{\Delta} R_2 = = 0.12, \ \overline{\Delta} R_3 = -0.33.$$

The impact of a rise in the total productivity of the industrial sector in the CR

We now change the total productivity of the CR's industrial sector in the following way: $A_{i1}: 1.2 \Rightarrow 1.3$. The effects on the education prices, wage rates and capital intensities are as

$$\overline{\Delta}\, p_1 = 6.69,\ \overline{\Delta}\, k_{i1} = \overline{\Delta}\, k_{e1} = \overline{\Delta}\, w_1 = 12.49,$$
$$\overline{\Delta}\, p_2 = \overline{\Delta}\, p_3 = \overline{\Delta}\, k_{i2} = \overline{\Delta}\, k_{i3} = \overline{\Delta}\, k_{e2} = \overline{\Delta}\, k_{e3} = \overline{\Delta}\, w_2 = \overline{\Delta}\, w_3 = 0.$$

The education price, wage rate, and capital intensities of the two sectors in the CR are increased, while the corresponding variables in the other two regions are not affected. The effects on the human capital, physical capital, and total labor input of regions are as follows

$$\overline{\Delta} H_1 = -8.59,\ \overline{\Delta} H_2 = 9.95,\ \overline{\Delta} H_3 = 9.90,\ \overline{\Delta} N_1 = 11.09,\ \overline{\Delta} N_2 = -9.62,$$
$$\overline{\Delta} N_3 = -9.48,\ \overline{\Delta} K_1 = 24.96,\ \overline{\Delta} K_2 = -9.63,\ \overline{\Delta} K_3 = -9.50,\ \overline{\Delta} F_{i1} = 15.37,$$
$$\overline{\Delta} F_{i2} = -9.60,\ \overline{\Delta} F_{i3} = -9.46,\ \overline{\Delta} F_{e1} = 11.72,\ \overline{\Delta} F_{e2} = -15.66,\ \overline{\Delta} F_{e3} = -15.48,$$
$$\overline{\Delta} n_{e1} = -4.63,\ \overline{\Delta} n_{e2} = -6.69,\ \overline{\Delta} n_{e3} = -6.63,\ \overline{\Delta} X_0 = -8.21.$$

The human capital level in the CR is decreased, but the labor force, capital stocks and output levels of the two sectors in the CR are increased. The human capital levels in the HR and the IR are increased, but the labor forces, capital stocks, and the output levels of the two sectors in the HR and IR are reduced. The population distribution, the levels of per capita GDP and the amenity levels are given as follows

$$\overline{\Delta} \overline{N}_1 = 19.33,\ \overline{\Delta} \overline{N}_2 = -16.08,\ \overline{\Delta} \overline{N}_3 = -15.92,\ \overline{\Delta} g_1 = -3.30,\ \overline{\Delta} g_2 = 7.69,$$
$$\overline{\Delta} g_3 = 7.64,\ \overline{\Delta} u_1 = 1.90,\ \overline{\Delta} u_2 = -2.63,\ \overline{\Delta} u_3 = -2.59.$$

The changes in the time distributions, consumption levels and levels of per capita wealth are

$$\overline{\Delta} T_1 = 0.04,\ \overline{\Delta} T_2 = -0.17,\ \overline{\Delta} T_3 = -0.18,\ \overline{\Delta} T_{h1} = \overline{\Delta} T_{h2} = \overline{\Delta} T_{h3} = 0,\ \overline{\Delta} T_{e1} = -0.11,$$
$$\overline{\Delta} T_{e2} = 0.50,\ \overline{\Delta} T_{e3} = 0.51,\ \overline{\Delta} c_1 = 4.69,\ \overline{\Delta} c_2 = 7.88,\ \overline{\Delta} c_3 = 7.84,\ \overline{\Delta} \bar{k}_1 = 4.69,$$
$$\overline{\Delta} \bar{k}_2 = 7.88,\ \overline{\Delta} \bar{k}_3 = 7.84.$$

The effects on the lot sizes and land rent are listed as follows

$$\overline{\Delta} l_1 = -9.99,\ \overline{\Delta} l_2 = 10.65,\ \overline{\Delta} l_3 = 10.47, \overline{\Delta} R_1 = 16.31,\ \overline{\Delta} R_2 = -2.50,\ \overline{\Delta} R_3 = -2.38.$$

5. Conclusions

This paper proposed an economic growth model of a multi-regional small open economy with endogenous human and physical capital accumulation in a perfectly competitive economy. The national economy consists of multiple regions and each region has one production sector and one education sector. Following the traditional literature of small open economies, it is assumed that the rate of interest is fixed in international market. The production side is the same as in the neoclassical growth theory. Different from, for instance, the Solow growth model and the Ramsey model, this study used a utility function, which determines saving, education and consumption with utility optimization without leading to a higher dimensional dynamic system like by the traditional approaches. Households move freely among regions, equalizing utility level among regions by choosing education, leisure, housing, goods and savings. A region's amenity is endogenous, depending on the region's output and population. The dynamics of *J*-regional national economy is controlled by a 2*J*-dimensional differential equations system. We simulated the motion of the model and examined effects of changes in some parameters. The model is built on many strict assumptions. It is possible to generalize the model in different directions. For instance, it is possible to use more general functional forms of utility and production functions. There are multiple production sectors and households are not homogenous. Any extension will cause some analytical difficulties because of the nature of regional dynamics. Moreover, it is known that issues related to tax competition among regions has increasingly caused great interests in economic geography.[25] It is also possible to extend the dynamic equilibrium framework proposed in this study to deal with these issues. As pointed out by Schultz (1963): "It would be surprising if there were not some major inefficiencies in the way resources are used in education, given the history of its growth, the changes in relative factor prices that have occurred, and the weak incentives that exist in much of education to adjust to changes in the prices of the resources employed… both private and public decisions affecting the allocation of resources to education are based on unnecessarily vague information. … These decisions, moreover, are made in an institutional setting that blunts private initiative and swamps public policy with other considerations." It is reasonable to consider other institutional factors in modeling education.

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[25] *See, for instance, Andersson and Forslid (2003), Baldwin and Krugman (2004), Bayindir-Upmann and Ziad (2005), Borck and Pflüger (2006).*

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Appendix: Proving Lemma 1

From (4), we have

$$k_{ij} = \left(\frac{\alpha_{ij} A_{ij}}{r^* + \delta_k} \right)^{1/\beta_{ij}}, \quad w_j = \beta_{ij} A_{ij} k_{ij}^{\alpha_{ij}}. \tag{A1}$$

From (6), we solve

$$k_{ej} = \frac{\alpha_{ej} w_j}{\beta_{ej} \left(r^* + \delta_k \right)}, \quad p_j = \frac{r^* + \delta_k}{\alpha_{ej} A_{ej} k_{ej}^{-\beta_{ej}}}. \tag{A2}$$

We see that k_{ij}, w_j, k_{ej}, and p_j are determined as functions of r^*, which is fixed in the international market.

From (3), we solve the labor distribution as functions of k

$$n_{ij} = \frac{\alpha_j k_{ij} - k_j}{\left(\alpha_j - 1 \right) k_{ij}}, \quad n_{ej} = \frac{k_j - k_{ij}}{\left(\alpha_j - 1 \right) k_{ij}}, \tag{A3}$$

where $\alpha_j \equiv k_{ej} / k_{ij}$. From $\bar{p}_j T_{ej} = \eta \bar{y}_j$ in (13) and the definition of $\bar{y}_j$, we have

$$T_{ej} = \varphi_{pj} \bar{k}_j + \varphi_{0j}, \tag{A4}$$

where

$$\varphi_{pj}\left(H_j \right) \equiv \frac{\left(1 + r^* \right) \eta}{\bar{p}_j}, \quad \varphi_{0j}\left(H_j \right) \equiv \frac{\eta T_0 \bar{w}_j}{\bar{p}_j}.$$

From (13), we have

$$T_{ej} + T_{hj} = \tilde{w}_j \bar{y}_j, \tag{A5}$$

where

$$\tilde{w}_j\left(k_i, H_j \right) \equiv \frac{\eta}{\bar{p}_j} + \frac{\sigma}{\bar{w}_j}.$$

Substituting (A5) into (8) yields

$$T_j = T_0 - \tilde{w}_j \bar{y}_j. \tag{A6}$$

From (A4) and (14)

$$\left(\varphi_{pj} \bar{k}_j + \varphi_{0j} \right) \bar{N}_j = F_{ej}. \tag{A7}$$

Insert $F_{ej} = A_{ej} k_{ej}^{\alpha_e} n_{ej} N_j$ and (A3) in (A7)

$$\left(\varphi_{pj} \bar{k}_j + \varphi_{0j}\right) \frac{\bar{N}_j}{A_{ej} k_{ej}^{\alpha_e}} = \frac{k_j - k_{ij}}{(\alpha - 1) k_{ij}} N_j .$$

Insert $N_j = T_j H_j^{m_j} \bar{N}_j$ and (A6) in the above equation

$$k_j = \Omega_j\left(H_j, \bar{k}_j\right) \equiv \frac{\left(\varphi_{pj} \bar{k}_j + \varphi_{0j}\right)\left(\alpha_j - 1\right) k_{ij}}{H_j^{m_j} A_{ej} k_{ej}^{\alpha_e} \left(T_0 - \tilde{w}_j \bar{y}_j\right)} + k_{ij} . \tag{A8}$$

From (A8) and (A3), we can treat k_j, n_{ij}, n_{ej}, $\bar{y}_j$, T_j, T_{ej}, and T_{hj} as functions of H_j and $\bar{k}_j$.

Substituting (12), (13) and $l_j \bar{N}_j = L_j$ into the utility functions, we have

$$U_j = \frac{\bar{\theta}_j \xi^{\xi_0} \lambda^{\lambda_0} \eta^{\eta_0} \sigma^{\sigma_0} L_j^{\gamma_0} F_{ij}^{a_i} F_{ej}^{a_e} \bar{N}_j^{b - \gamma_0} \bar{y}_j^{\xi_0 + \lambda_0 + \eta_0 + \sigma_0}}{\bar{p}_j^{\eta_0} \bar{w}_j^{\sigma_0}} . \tag{A9}$$

From (4) and (6), it is straightforward to show

$$F_{ij} = \bar{n}_{ij} N_j , \quad F_{ej} = \bar{n}_{ej} N_j , \tag{A10}$$

where

$$\bar{n}_{ij}\left(H_j, \bar{k}_j\right) \equiv \frac{w_j n_{ij}}{\beta_{ij} A_{ij}}, \quad \bar{n}_{ej}\left(H_j, \bar{k}_j\right) \equiv \frac{w_j n_{ej}}{p_j \beta_{ej} A_{ej}} .$$

Insert (A10) in (A9)

$$U_j = u_j \xi^{\xi_0} \lambda^{\lambda_0} \eta^{\eta_0} \sigma^{\sigma_0} \bar{N}_j^{B} , \tag{A11}$$

where we also use $N_j = T_j H_j^{m_j} \bar{N}_j$ and

$$u_j\left(H_j, \bar{k}_j\right) \equiv \frac{\bar{\theta}_j L_j^{\gamma_0} \bar{n}_{ij}^{a_i} \bar{n}_{ej}^{a_e} \left(T_j H_j^{m_j}\right)^{a_i + a_e} \bar{y}_j^{\xi_0 + \lambda_0 + \eta_0 + \sigma_0}}{\bar{p}_j^{\eta_0} \bar{w}_j^{\sigma_0}} .$$

Inserting (A11) in equations (14), we get

$$n_j\left(H_j, \bar{k}_j, H_1, \bar{k}_1\right) = \left(\frac{u_1}{u_j}\right)^{1/B}, \quad j = 2, \cdots, J , \tag{A12}$$

where we require $B \neq 0$. From (21), we have

$$\bar{N}_1\left(\left(H_j\right), \left(\bar{k}_j\right)\right) = \frac{\bar{N}}{1 + \sum_{j=2}^{J} n_j}, \tag{A13}$$

where $\left(H_j\right) \equiv \left(H_1, \cdots, H_J\right)$ and $\left(\bar{k}_j\right) \equiv \left(\bar{k}_1, \cdots, \bar{k}_J\right)$. From this equation and equations (A1), we determine the population distribution as functions of $\hat{y}_j$ as follows

$$\bar{N}_j\left(\left(H_j\right), \left(\bar{k}_j\right)\right) = n_j \bar{N}_1, \quad j = 2, \dots, J. \tag{A14}$$

It is straightforward to confirm that all the variables can be expressed as functions of $\left(\bar{k}_j\right)$ and $\left(H_j\right)$ by the following procedure: $\bar{N}_j$ by (A13) and (A14) → k_{ij} and w_j by (A1) → p_j and k_{ej} by (A2) → F_{ij} and F_{ej} by (A10) → $\bar{y}_j$ by (10) → c_j, s_j, T_{ij} and T_{ej} by (13) → $l_j = L_j / \bar{N}_j$ → $R_j = \xi \bar{y}_j / l_j$ → T_j by (A6) → $N_j = T_j H_j^{m_j} \bar{N}_j$ → k_j by (A8) → $C_j = c_j \bar{N}_j$ → n_{ij} and n_{ej} by (A3) → $N_{ij} = n_{ij} N_j$ and $N_{ej} = n_{ej} N_j$ → $K_{ij} = k_{ij} N_{ij}$ and $K_{ej} = k_{ej} N_{ej}$ → K by (A18) → $\bar{K}$ by (19) → U_j by (A11). From this procedure, (15) and (16), we have

$$\dot{\bar{k}}_j = \Lambda_j\left(\left(\bar{k}_j\right), \left(H_j\right)\right) \equiv \lambda \bar{y}_j - \bar{k}_j,$$

$$\dot{H}_j = \Psi_j\left(\left(\bar{k}_j\right), \left(H_j\right)\right) \equiv \frac{\upsilon_{ej} F_{ej}^{a_{ej}} \left(H_j^{m_j} T_{ej} \bar{N}_j\right)^{b_{je}}}{H_j^{\pi_{ej}} \bar{N}_j} + \frac{\upsilon_{ij} F_{ij}^{a_{ij}}}{H_j^{\pi_{ij}} \bar{N}_j} + \frac{\upsilon_{hj} C_j^{a_{hj}}}{H_j^{\pi_{hj}} \bar{N}_j} - \delta_{jh} H_j. \tag{A15}$$

We have thus proved Lemma 1.